Math 213 Fall 2011
TA: Mushfeq Khan

Practice Problems for Midterm II

1. Find the volume of the solid of revolution formed by rotating the region bounded by the curves $y = 0$, $x = 1$ and $y = \sqrt{x}$ about the x-axis.

2. Let $f(x) = 1000x$ be the rate of money flow in dollars per year. Assume a 10 year period of 10% interest rate compounded continuously. Compute the following (you can leave your answers in terms of e):

   (a) The total money flow.

   (b) The present value of the money.

3. For each of the following improper integrals, either evaluate it or explain why it diverges:

   (a)
   $$\int_2^\infty (x^2+1)e^{-(x^3+3x)}\,dx$$

   (b)
   $$\int_{-\infty}^2 (x^2+1)e^{-(x^3+3x)}\,dx$$

   (c)
   $$\int_2^\infty \frac{\ln x}{x^3}\,dx$$

   (d)
   $$\int_0^\infty \frac{dx}{(2x+1)^5}$$

   (e)
   $$\int_{-\infty}^{-1} xe^{-x^2}\,dx$$

   (f)
   $$\int_1^\infty \frac{(\ln x)^2}{x^2}\,dx$$

4. Suppose
   $$f(x,y) = \frac{xy^2}{x+y}.$$
   Find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.

5. If $f(x,y) = e^{x^2+y^2}$, then what is $f_{xy}(1,1)$?

6. For each of the following functions, find all the critical points. Determine if they are points for relative maximum, relative minimum or saddle points.

   (a) $f(x, y) = 2x^3 + y^2 - 6xy$

   (b) $f(x, y) = y^3 + x^2 - 2xy - 6x + 5y - 3$

   (c) $f(x, y) = x^2 + 4y^3 - 6xy - 1$

   (d) $f(X, y) = 3x^2 + 7y^3 - 42xy + 5$

7. The profit from the sale of x units of radiators for automobiles and y units of radiators for generators is given by:

$$P(x, y) = -x^2 - y^2 + 4x + 8y$$

   Find values of x and y that maximize profit if the firm must produce a total of 6 units of radiators.

8. Find positive numbers x and y, whose sum is 75, such that xy^2 is maximized.

9. An ant is walking on a circular wire on the xy-plane satisfying the equation $x^2 + y^2 = 1$. Suppose that the temperature at (x, y) on the xy-plane is given by $f(x, y) = x^2 + xy + y^2$ in hundreds of degrees Fahrenheit (ouch). Then where are the coolest places *on the wire* that the ant can stay?

10. Find the longest and shortest distances from the curve $x^2 + xy + y^2 = 1$ to the origin using the method of Lagrange multipliers. (Hint: minimize the function $x^2 + y^2$ subject to the constraint $x^2 + xy + y^2 = 1$.)