5.3 # 13, 16, 18, 30, 34, 36, 38, 44, 50, 70 & Additional

13.

x	1	10	10^2	10^3	10^{-1}	10^{-2}	10^{-3}
$\log_{10} x$	0	1	2	3	-1	-2	-3

16. (a) $1 < \log_9 80 < 2$

$\ln(e^2) = 2$ ← larger.

(b) $1 < \log_2 3$ ← larger

$\log_3 2 < 1$

18. (a) $\log_{25}\left(\frac{1}{625}\right) = -2$

(b) $\log_{16}\left(\frac{1}{64}\right) = -\frac{3}{2}$

(c) $\log_{10} 10 = 1$

(d) $\log_2 8\sqrt{2} = 3 + \frac{1}{2} = \frac{7}{2}$

30. $y = \ln(-x) + e$ Domain $(-\infty, 0)$

Range $(-\infty, \infty)$

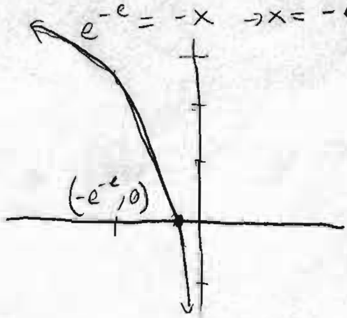
y-int: none

Vertical Asymptote $x = 0$

x-int $0 = \ln(-x) + e$

$-e = \ln(-x)$

$e^{-e} = -x \rightarrow x = -e^{-e} = -\frac{1}{e^e}$



34. $10^{2x-1} = 145$

$2x - 1 = \log_{10} 145$

$x = \frac{1 + \log_{10} 145}{2}$

36. $(10^x)^2 = 40$

$10^{2x} = 40$

$2x = \log_{10} 40$

$x = \frac{1}{2} \log_{10} 40$

38. $e^{t-1} = 16$

$t - 1 = \ln 16$

$t = 1 + \ln 16$

44.

Cali:

$5.2 = \log_{10}\left(\frac{A}{A_0}\right)$

$10^{5.2} = \frac{A}{A_0} \rightarrow A = A_0 10^{5.2}$

Indonesia:

$8.0 = \log_{10} \frac{A}{A_0}$

$10^8 = \frac{A}{A_0} \rightarrow A = A_0 10^8$

$\frac{A_0 10^8}{A_0 10^{5.2}} = 10^{8-5.2} = 10^{2.8}$ times stronger

50. $[H^+] = 3.5 \times 10^{-9}$

$pH = -\log_{10}(3.5 \times 10^{-9})$

$= -\log_{10}(3.5) - \log_{10} 10^{-9} = \underbrace{-\log_{10}(3.5)}_{\leq 1} + 9$

So this is a base.

70. Domain of $\ln(\ln(\ln x))$

① $\ln(x)$ needs $x > 0$

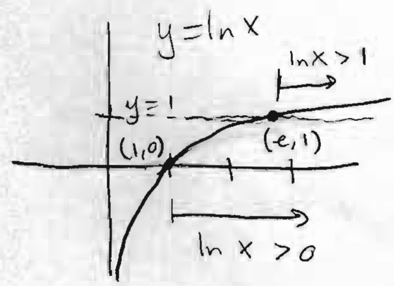
② $\ln(\ln(x))$ needs $\ln(x) > 0$

$\Rightarrow x > 1$

③ $\ln(\ln(\ln(x)))$ needs $\ln(\ln(x)) > 0$

$\Rightarrow \ln(x) > 1$

$\Rightarrow x > e$



(e, ∞)

5.3 Additional Problems

1. (a) $f(g(x)) = e^{2(\ln(x))} = e^{\ln(x^2)} = x^2$

$g(f(x)) = \ln(e^{2x}) = 2x$

(b) $f(g(x)) = 3\ln(e^x) = 3x \quad g(f(x)) = e^{3\ln(x)} = e^{\ln x^3} = x^3$

(c) $f(g(x)) = e^{\ln(x-3)+3} = e^{\ln(x-3)}e^3 = e^3(x-3) \quad g(f(x)) = \ln(e^{x+3} - 3)$

f^{-1}: $y = e^{x+3}$
$x = e^{y+3}$
$\ln x = y+3$
$y = -3+\ln x$

$f^{-1}(x) = -3+\ln x$

g^{-1}: $y = \ln(x-3)$
$x = \ln(y-3)$
$e^x = y-3$
$y = e^x + 3$

$g^{-1}(x) = e^x + 3$

(d) $f(g(x)) = e^{\ln(x-3)} + 3 = (x-3)+3 = x \quad g(f(x)) = \ln(e^x + 3 - 3) = \ln(e^x) = x$

so $f(x)$ and $g(x)$ are inverses

2. (a) $y = e^{2x+3}$
$x = e^{2y+3}$
$\ln(x) = 2y+3$
$\frac{\ln(x) - 3}{2} = y$
$f^{-1}(x) = \frac{1}{2}\ln(x) - \frac{3}{2}$

(b) $y = e^{3x}+2$
$x = e^{3y}+2$
$x-2 = e^{3y}$
$\ln(x-2) = 3y$
$y = \frac{1}{3}\ln(x-2)$
$f^{-1}(x) = \frac{1}{3}\ln(x-2)$

(c) $y = 1-\ln(2x)$
$x = 1-\ln(2y)$
$x-1 = -\ln(2y)$
$-x+1 = \ln(2y)$
$e^{-x+1} = 2y$

$y = \frac{1}{2}e^{-x+1}$
$f^{-1}(x) = \frac{1}{2}e^{-x+1}$

3. (a) note: not a function to start.
$x^2 = \ln(y^3+2)$
$e^{x^2} = y^3+2$
$e^{x^2} - 2 = y^3$
$y = \sqrt[3]{e^{x^2} - 2}$
"inverse"

(b) $x^3 = 1+\log_{10}(3y+7)$
$x^3 - 1 = \log_{10}(3y+7)$
$10^{x^3-1} = 3y+7$
$10^{x^3-1} - 7 = 3y$
$y = \frac{10^{x^3-1} - 7}{3}$
inverse

(c) $y = e^{e^x}$
$x = e^{e^y}$
$\ln x = e^y$
$\ln(\ln x) = y$

5.4 # 2, 4, 6, 10, 12, 15, 18, 20, 28, 30, 38, 44, 48, 62, 63, 72b, 76.

2. $\log_{10}(40) + \log_{10}\left(\frac{5}{2}\right)$

$= \log_{10}\left(40 \cdot \frac{5}{2}\right)$

$= \log_{10}(100) = 2$

4. $\log_9(25) - \log_9(75) = -\frac{1}{2}$

6. $\ln e^3 - \ln e = 2$

0. $\log_b b^b = b$

12. $2\log_{10} x - 3\log_{10} y$

$\log_{10}\left(\frac{x^2}{y^3}\right)$

15. (a) $\ln 3 - 2\ln 4 + \ln 32$

$\ln 3 - \ln 16 + \ln 32$

$\ln 6$

(b) $\ln 3 - 2(\ln 4 + \ln 32)$

$\ln 3 - 2\ln 2^7$

$\ln 3 - \ln 2^{14}$

$\ln \frac{3}{2^{14}} = \ln \frac{3}{16384}$

18. $\ln(x^3-1) - \ln(x^2+x+1)$

$= \ln(x-1)$

20. (a) $\log_{10}\sqrt{(x+1)(x+2)}$

$= \frac{1}{2}\log_{10}(x+1) + \frac{1}{2}\log_{10}(x+2)$

(b) $\ln\sqrt{\frac{(x+1)(x+2)}{(x-1)(x-2)}}$

$= \frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x+2) - \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x-2)$

28 & 30 $\log_b 2 = A$ $\log_b 3 = B$ $\log_b 5 = C$

28 (a) $\log_b 10 = A + C$

(b) $\log_b 100 = 2A + 2C$

(c) $\log_b(.01) = -2A - 2C$

(d) $\log_b(.03) = B - 2A - 2C$

30. (a) $\log_b \sqrt{5} = \frac{1}{2}C$

(b) $\log_b \sqrt{15} = \frac{1}{2}B + \frac{1}{2}C$

(c) $\log_b \sqrt[3]{.4} = \frac{1}{3}A - \frac{1}{3}C$

(d) $\log_b \sqrt[4]{60} = \frac{1}{4}(C + 2A + B)$

38. $\log_{10} A = a$ $\log_{10} B = b$ $\log_{10} C = c$

(a) $\log_{10} A + 2\log_{10}\left(\frac{1}{A}\right)$

$= a - 2a = -a$

(b) $\log_{10}\left(\frac{A}{10}\right) = a - 1$

(c) $\log_{10} \frac{100A^2}{B^4\sqrt[3]{C}}$

$= 2 + 2a - 4b - \frac{1}{3}c$

(d) $\log_{10}\left(\frac{(4B)^5}{C}\right)$

$= 5\log_{10} 4 + 5b - c$

44. $3e^{1+t} = 2$

$e^{1+t} = \frac{2}{3}$

$1 + t = \ln\left(\frac{2}{3}\right)$

$t = -1 + \ln\left(\frac{2}{3}\right)$

48. $10^{2x+3} = 280$

$(2x+3)\ln 10 = \ln(280)$

$2x + 3 = \frac{\ln(280)}{\ln 10}$

$x = \frac{1}{2}\left(-3 + \frac{\ln(280)}{\ln(10)}\right)$

62.

(a) $\log_b(x+y) \neq \log_b x + \log_b y$

b/c $3 = \log_2(4+4) \neq \log_2 4 + \log_2(4) = 4$

(b) $\frac{\log_b x}{\log_b y} \neq \log_b x - \log_b y$

(c) $(\log x)(\log y) \neq \log x + \log y$

(d) $(\log x)^k \neq k \log x$

63. a) true
b) true
c) true
d) F
e) T
f) F
g) T
h) F
i) T
j) F
k) F
l) T
m) T

72 (b) optional.

prove $\log_a x = \frac{\log_b x}{\log_b a}$

76. is there k s.t.

$e^x = 2^{kx}$ $\forall x$?

$\ln e^x = \ln(2^{kx})$

$x \ln e = kx \ln 2$

$x = kx \ln 2$ (if $x \neq 0$, can divide by it)

$1 = k \ln 2$

$\frac{1}{\ln 2} = k$

if $x = 0$

$e^0 \overset{?}{=} 2^{k(0)}$ ✓

So works for all x

5.5 # 2, 4, 6, 8, 10, 12, 26, 28, 30, 32, 34, 55, 58, 63, 75, 78, 80, 94

5.6 # 2, 4, 6, 17, 20, 21

27

2) $7^{-4x} = 2^{1+3x}$

$\ln(7^{-4x}) = \ln(2^{1+3x})$

$-4x\ln 7 = (1+3x)\ln 2$

$-4x\ln 7 = \ln 2 + 3x\ln 2$

$-4x\ln 7 - 3x\ln 2 = \ln 2$

$x = \dfrac{\ln 2}{-4\ln 7 - 3\ln 2}$

4) $\log_3(\log_3(2x)) = -2$

$\log_3(2x) = 3^{-2}$

$2x = 3^{1/9}$

$x = \left(\frac{1}{2}\right)3^{1/9}$

6) $\log_2(2x^2 - 4) = 5$

$2x^2 - 4 = 32$

$2x^2 = 36$

$x^2 = 18$

$x = \pm 3\sqrt{2}$

8) $\log_9(x^2+x) = .5$

$x^2 + x = 3$

$x^2 + x - 3 = 0$

$x = \dfrac{-1 \pm \sqrt{1+4(3)}}{2}$

$= \dfrac{-1 \pm \sqrt{13}}{2}$

10) $3(2^{2x}) - 11(2^x) - 4 = 0$

$3y^2 - 11y - 4 = 0$

$(3y+1)(y-4) = 0$

$y = -\frac{1}{3}$ $y = 4$

$2^x = -\frac{1}{3}$ $2^x = 4$

no soln. $x = 2$

12) (a) $(\log_{10}x)^2 = 2\log_{10}x$

$(\log_{10}x)^2 - 2\log_{10}x = 0$

$\log_{10}x(\log_{10}x - 2) = 0$

$\log_{10}x = 0$ or $\log_{10}x - 2 = 0$

$x = 1$ $\log_{10}x = 2$

$x = 10^2$

(b) $\log_{10}(x^2) = 2\log_{10}x$

$2\log_{10}(x^2) = 2\log_{10}x$

for all x in Domain, i.e. $x > 0$

26) (a) $4e^{6x} - 12e^{3x} + 9 = 0$

$(2e^{3x} - 3)(2e^{3x} - 3) = 0$

$(2e^{3x} - 3)^2 = 0$

$2e^{3x} - 3 = 0$

$2e^{3x} = 3$

$e^{3x} = \frac{3}{2}$

$3x = \ln\left(\frac{3}{2}\right)$

$x = \frac{1}{3}\ln\left(\frac{3}{2}\right)$

(b) $4e^{6x} + 12e^{3x} + 9 = 0$

$(2e^{3x} + 3)^2 = 0$

$2e^{3x} = -3$

no solutions.

(c) $4e^{6x} - 16e^{3x} - 9 = 0$

$y = e^{3x}$

$4y^2 - 16y - 9 = 0$

$y = \dfrac{16 \pm \sqrt{256 - 4(4)(-9)}}{8}$

$= \dfrac{16 \pm 20}{8}$

$y = \frac{9}{2}$ or $y = -\frac{1}{2}$

$e^{3x} = \frac{9}{2}$ $e^{3x} = -\frac{1}{2}$ no solution

$3x = \ln\frac{9}{2}$

$\boxed{x = \frac{1}{3}\ln\frac{9}{2}}$

(d) $e^{6x} - 12e^{3x} - 9 = 0$

$y^2 - 12y - 9 = 0$

$y = \dfrac{12 \pm \sqrt{144 - 4(1)(-9)}}{2}$

$y = \dfrac{12 \pm 6\sqrt{5}}{2}$

$e^{3x} = \dfrac{12 + 6\sqrt{5}}{2}$ $e^{3x} = \dfrac{12 - 6\sqrt{5}}{2}$ no solution

$3x = \ln\left(\frac{12+6\sqrt{5}}{2}\right)$

$x = \frac{1}{3}\ln\left(\frac{12+6\sqrt{5}}{2}\right) = \frac{1}{3}\ln(6+3\sqrt{5})$

28) $e^x + e^{-x} = 2$

$e^{2x} + 1 = 2e^x$

$e^{2x} - 2e^x + 1 = 0$

$(e^x - 1)^2 = 0$

$e^x = 1$

$x = 0$

30) $e^{3x} = 10^{2x}2^{1-x}$

$3x = \ln(10^{2x}2^{1-x})$

$3x = \ln(10^{2x}) + \ln(2^{1-x})$

$3x = 2x\ln(10) + \underbrace{(1-x)\ln 2}_{\ln 2 - x\ln 2}$

$3x - 2x\ln 10 + x\ln 2 = \ln 2$

$x(3 - 2\ln 10 + \ln 2) = \ln 2$

$x = \dfrac{\ln 2}{3 - 2\ln 10 + \ln 2}$

32) $\log_6 x + \log_6(x+1) = 0$

$\log_6(x^2 + x) = 0$

$x^2 + x = 1$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(-1)}}{2}$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$

~~$x = \frac{-1-\sqrt{5}}{2}$~~

$x = \dfrac{-1+\sqrt{5}}{2}$

34) $\log_2(x+4)=2-\log_2(x+1)$

$\log_2((x+4)(x+1))=2$

$x^2+5x+4=4$

$x^2+5x=0$

$x(x+5)=0$

$x=0$ or $\cancel{x=-5}$

55) $\ln(2-5x)>2$

$2-5x>e^2$

$-5x>e^2-2$

$x<\frac{e^2-2}{-5}$

58) $4^{5-x}>15$

$5-x>\log_4 15$

$-x>-5+\log_4 15$

$x<5-\log_4 15$

63) $e^{x^2-4x}\geq e^5$

$x^2-4x\geq 5$

$x^2-4x-5\geq 0$

$(x-5)(x+1)\geq 0$

+ − +

-1 5

$(-\infty,-1]\cup[5,\infty)$

75) $3(\ln x)^2-2\ln x-8=0$

$(3\ln x+4)(\ln x-2)=0$

$3\ln x+4=0$ or $\ln x-2=0$

$3\ln x=-4$ $\quad \ln x=2$

$\ln x=-\frac{4}{3}$ $\quad x=e^2$

$x=e^{-4/3}$ or $x=e^2$

78) $\frac{\ln(\sqrt{x+4}+2)}{\ln\sqrt{x}}=2$

$\ln(\sqrt{x+4}+2)=2\ln(\sqrt{x})$

$=\ln(\sqrt{x})^2$

$\ln(\sqrt{x+4}+2)=\ln x$

$\sqrt{x+4}+2=x$

$\sqrt{x+4}=x-2$

$x+4=x^2-4x+4$

78 cont'd)

$0=x^2-5x$

$=x(x-5)$

so

$x=0$ or $x=5$

Check:

$\cancel{\frac{\ln(\sqrt{4}+2)}{\ln(0)}\stackrel{?}{=}2}$

$\frac{\ln(\sqrt{5+4}+2)}{\ln\sqrt{5}}\stackrel{?}{=}2$

$\frac{\ln 5}{\ln\sqrt{5}}\stackrel{?}{=}2$

$\log_{\sqrt{5}}5\stackrel{?}{=}2$

$5\stackrel{?}{=}(\sqrt{5})^2$ ✓

80) $3\ln x=\alpha+3\ln\beta$

$\ln x=\frac{\alpha}{3}+\ln\beta$

$x=e^{\frac{\alpha}{3}+\ln\beta}$

$=e^{\frac{\alpha}{3}}e^{\ln\beta}$

$x=\beta e^{\alpha/3}$

94) $(\pi x)^{\log_{10}\pi}=(ex)^{\log_{10}e}$

$(\pi x)^{\log_{10}\pi}-(ex)^{\log_{10}e}=0$

$x^{\log_{10}e}\left(\pi^{\log_{10}\pi}x^{\log_{10}\pi-\log_{10}e}-e^{\log_{10}e}\right)=0$

$x^{\log_{10}e}=0$ or $\pi^{\log_{10}\pi}x^{\log_{10}\pi-\log_{10}e}-e^{\log_{10}e}=0$

$x=0$ $\quad \pi^{\log_{10}\pi}x^{\log_{10}\pi-\log_{10}e}=e^{\log_{10}e}$

$x^{\log_{10}\pi-\log_{10}e}=\frac{e^{\log_{10}e}}{\pi^{\log_{10}\pi}}$

or

$$x=\left(\frac{e^{\log_{10}e}}{\pi^{\log_{10}\pi}}\right)^{\frac{1}{\log_{10}\left(\frac{\pi}{e}\right)}}$$

$\left(x=\frac{1}{\pi e}\right)$

5.6

2) $A(t)=1000(1+.055)^t$

$2500=1000(1.055)^t$

$2.5=1.055^t$

$\ln(2.5)=t\ln 1.055$

$\frac{\ln(2.5)}{\ln(1.055)}=t$

$t\approx 17.1$

So after 18 years will exceed \$2500

4) $10000=P(1+.07)^{10}$

$P=\frac{10000}{(1.07)^{10}}$

$(P\approx 5083.49)$

6) (a) $A=3000(1+.06)$

$=3180$

(b) $A=3000\left(1+\frac{.06}{2}\right)^2$

$=3182.70$

(c) $A=3000\left(1+\frac{.06}{365}\right)^{365}$

≈ 3185.49

17) $5000=Pe^{(.065)10}$

$P=\frac{5000}{e^{.65}}$

(≈ 2610.23)

20) 1st

$A=1000\left(1+\frac{.0525}{2}\right)^{2(12)}$

$\approx \$1862.41$

2nd

$A=1000e^{(.0525)12}$

≈ 1877.61

2nd makes ≈ \$15.20 more

21) 1st

$A(5)=10000(1+.06)^5$

≈ 13382.26

2nd

$A(5)=10000e^{(.05)5}$

≈ 12840.25

1st is better investment.