P6

Section 5.7 # 1, 8, 13, 16, 25, 31, 35

1. $N(t) = 2000\, e^{kt}$

$3800 = 2000\, e^{k(2)}$

$1.9 = e^{2k}$

$\ln(1.9) = 2k$

$k = \frac{1}{2}\ln(1.9)\ (\approx .3209)$

b) $N(5) = 2000\, e^{\frac{1}{2}\ln(1.9)5}$

$= 2000\, e^{\frac{5}{2}\ln(1.9)}$

$= 2000\,(1.9)^{5/2}$

(≈ 9952)

c) When will pop. reach 10000?

$10000 = 2000\, e^{\frac{1}{2}\ln(1.9)t}$

$5 = (1.9)^{\frac{t}{2}}$

$\log_{1.9} 5 = \frac{t}{2}$

$2\log_{1.9} 5 = t$

$(t \approx 5.015)$

8. a) Mali: $N(15) = 11.2\, e^{(.031)15}$

≈ 17.8 Million

Cuba: $N(15) = 11.1\, e^{(.007)15}$

≈ 12.3 Million

b) $20 = 11.2\, e^{(.031)t}$

$\frac{20}{11.2} = e^{.031t}$

$\ln\left(\frac{20}{11.2}\right) = .031t$

$t = \frac{1}{.031}\ln\left(\frac{20}{11.2}\right)$

≈ 18.7 years.

Cuba: $N(18.7) \approx 11.1\, e^{(.007)(18.7)}$

≈ 12.7 Million.

13. a) $2 = e^{.006t}$

$t_{double} = \frac{1}{.006}\ln 2 \approx 115.5$ years

b) $2 = e^{.016t}$ $\quad t \approx 43.3$ years

c) $2 = e^{.026t}$ $\quad t \approx 26.7$ years

d) $2 = e^{.037t}$ $\quad t \approx 18.7$ years.

16. (a) $368.37 = 316.75\, e^{k39}$

$\frac{1}{39}\ln\left(\frac{368.37}{316.75}\right) = k \approx .00387$

(b) $N(15) = 316.75\, e^{k(15)}$

≈ 348.94 too high

% error $\approx .9\%$

(c) $N(30) \approx 355.76$ too high

% error $\approx .5\%$

$N(35) \approx 362.71$ too high

% error $.5\%$

d) It was used to find k.

25. $k = \frac{-\ln 2}{8}$

$N(7) = 1 \cdot e^{\frac{-\ln 2}{8}\cdot 7} \approx .545$ g.

31. $k = \frac{-\ln 2}{13}$

(a) $N(5) = 2e^{\left(\frac{-\ln 2}{13}5\right)} \approx 1.53$ g

(b) $.1(2) = 2e^{\left(\frac{-\ln 2}{13}\right)t}$

$\ln(.1) = \frac{-\ln 2}{13}t$

$t = \frac{13\ln(.1)}{-\ln 2} \approx 43.2$ years.

35. (a) $k = \frac{-\ln 2}{28}$

(b) $\frac{1}{1000} = e^{\frac{-\ln 2}{28}t}$

$\frac{28\ln\left(\frac{1}{1000}\right)}{-\ln 2} = t \approx 279$ years.

(c) $2^{10} = 1024 \approx 1000$

So about 10 half lives

≈ 280 years

Section 7.1 #2, 7, 10, 12, 18, 24, 30, 38, 40, 46, 48, 54, 56, 58, 60, 62, 64, 65, 72, 75

2. $1+3i+5+4-2i+i$

$10+2i$

7. (a) $14-4i$ (b) $-4-8i$

10. (a) $4+14i-14i-49i^2 = 53$

(b) $\frac{-1+3i}{2+7i}\left(\frac{2-7i}{2-7i}\right) = \frac{-2+6i+7i-21i^2}{53} = \frac{19+13i}{53}$

(c) $\frac{1}{2+7i}\left(\frac{2-7i}{2-7i}\right) = \frac{2-7i}{53}$

(d) $\frac{1}{2+7i}\cdot(-1+3i) = \frac{-1+3i}{2+7i} = \frac{19+13i}{53}$

12. (a) $2-3i+9+4i = 11+i$

(b) $(\overline{11-i}) = 11+i$

(c) $-8i$

18. $9^2+4^2 = 81+16 = 97$

24. $(-7+7i)(11-i) = -77+77i+7i-7i^2$

$= -70+84i$

30. $\frac{9-4i}{2+3i} = \frac{(9-4i)(2-3i)}{4+9}$

$= \frac{18-8i-27i+12i^2}{13}$

$= \frac{6-35i}{13}$

38. $\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right) = \frac{1-i\sqrt{3}-i\sqrt{3}+i^2\sqrt{3}^2}{1+(\sqrt{3})^2}$

$= \frac{-2-2i\sqrt{3}}{4} = -\frac{1}{2}-\frac{i\sqrt{3}}{2}$

40. $\frac{i+i^2}{i^3+i^4} = \frac{i+i^2}{i^2(i+i^2)} = \frac{1}{i^2} = \frac{1}{-1} = -1$

46. $\sqrt{-25}+i = 5i+i = 6i$

48. $\sqrt{-4}\,\sqrt{4} = (2i)(2i) = -4$

54. (a) $d = (-6)^2-4(12)(1) = 36-48 = -12$

(b) $x = \frac{+6\pm\sqrt{-12}}{2} = \frac{6\pm 2i\sqrt{3}}{2} = 3\pm i\sqrt{3}$

56. $d = 4^2-4(-2)(-10) = 16-80 = -64$

$z = \frac{-4\pm\sqrt{-64}}{2(-10)} = \frac{-4\pm i8}{-20} = \frac{-1\pm 2i}{-5}$

$z = \frac{1}{5}-\frac{2}{5}i$ or $z = \frac{1}{5}+\frac{2}{5}i$

58. $d = 49-4(5)(3) = 49-60 = -11$

$z = \frac{7\pm\sqrt{-11}}{2(3)} = \frac{7\pm i\sqrt{11}}{6}$

60. $d = 4-4(\frac{1}{2})(\frac{9}{4}) = \frac{8}{2}-\frac{9}{2} = -\frac{1}{2}$

$z = \frac{-2\pm\sqrt{-\frac{1}{2}}}{2(\frac{1}{2})} = -2\pm i\sqrt{\frac{1}{2}}$

62. (a) $x = \frac{2\pm\sqrt{4-4(5)}}{2}$

$= \frac{2\pm\sqrt{-16}}{2} = \frac{2\pm 4i}{2} = 1\pm 2i$

(b) $(1+2i)(1-2i) = 1-2i+2i-4i^2 = 5$

(c) $c=5$ ✓

64. $\left(\frac{-1+i\sqrt{3}}{2}\right)^2+\left(\frac{-1-i\sqrt{3}}{2}\right)^2 = -1$

$\frac{1}{4}(1-i\sqrt{3}-i\sqrt{3}+3i^2)+\frac{1}{4}(1+i\sqrt{3}+i\sqrt{3}+3i^2)$

$= \frac{1}{4}(-2-2i\sqrt{3})+\frac{1}{4}(-2+2i\sqrt{3}) = \frac{1}{4}(-4) = -1$ ✓

65. $z = \frac{-1+i\sqrt{3}}{2}$ $w = \frac{-1-i\sqrt{3}}{2}$

(a) $z^3 = \left(\frac{-1-i\sqrt{3}}{2}\right)\left(\frac{-1+i\sqrt{3}}{2}\right) = \left(\frac{1+3}{4}\right) = 1$ ✓

$w^3 = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right) = 1$

(b) $z \cdot w = 1$

(c) $z^2 = w$ ✓ $w^2 = z$ ✓

(d) $(1-z+w)(1+z-w)$

$= \left(\frac{2+1-i\sqrt{3}-1-i\sqrt{3}}{2}\right)\left(\frac{2-1+i\sqrt{3}+1+i\sqrt{3}}{2}\right)$

$= \left(\frac{2-2i\sqrt{3}}{2}\right)\left(\frac{2+2i\sqrt{3}}{2}\right) = (1-i\sqrt{3})(1+i\sqrt{3}) = 4$ ✓

72. $2x^2(x+2)+3(x+2) = (2x^2+3)(x+2) = 0$

$x = -2$ or $2x^2+3=0 \rightarrow x^2 = -\frac{3}{2}$ $x = \pm i\sqrt{3}$

75. $\left(\frac{a+bi}{a-bi}\right)\left(\frac{a+bi}{a+bi}\right)+\left(\frac{a-bi}{a+bi}\right)\left(\frac{a-bi}{a-bi}\right)$

$= \frac{a^2+2abi-b^2+a^2-2abi-b^2}{a^2+b^2}$

$= \frac{2a^2-2b^2}{a^2+b^2}$ real

imaginary part $= 0$