

6.2 # 2, 4, 6, 14, 18, 20, 22, 34a & b, 38, 40

$$2. \begin{cases} -3x + 7y + 2z = -19 \\ y + z = 1 \\ -2z = -2 \\ \hookrightarrow z = 1 \\ \hookrightarrow y = 0 \\ \hookrightarrow -3x + 2 = -19 \\ -3x = -21 \\ x = 7 \end{cases}$$

$(7, 0, 1)$

$$4. \begin{cases} 2x + 7z = -4 \\ 5y - 3z = 6 \\ 6z = 18 \rightarrow z = 3 \\ \hookrightarrow 5y - 9 = 6 \rightarrow 5y = 15 \\ y = 3 \end{cases}$$

$$2x + 7(3) = -4 \\ 2x = -25 \quad x = -\frac{25}{2}$$

$$6. \begin{cases} 3x - 2y + z = 4 \\ 3z = 9 \rightarrow z = 3 \\ 3x - 2y = 1 \rightarrow 2y = 3x - 1 \\ \left(x, \frac{3x-1}{2}, 3\right) \end{cases}$$

$$14. \begin{cases} x + 2z = 5 \\ y - 3z = -16 \\ x - 2y + 4z = 8 \\ \begin{cases} x + 2z = 5 \\ y - 3z = -16 \\ -2y + 2z = 3 \end{cases} \end{cases}$$

$$\begin{cases} x + 2z = 5 \\ y - 3z = -16 \\ -5z = -24 \\ \hookrightarrow z = \frac{24}{5} \\ \hookrightarrow y = -1 \end{cases}$$

$$\hookrightarrow x = 4$$

$$18. \begin{cases} 7x + 5y - 7z = -10 \\ 2x + y + z = 7 \\ x + y - 3z = -8 \\ \begin{cases} 21x + 12y = 39 \\ 2x + y + z = 7 \\ 7x + 4y = 13 \end{cases} \\ \begin{cases} 2x + y + z = 7 \\ 7x + 4y = 13 \end{cases} \rightarrow 4y = 13 - 7x \end{cases}$$

$$2x + \frac{13-7x}{4} + z = 7 \rightarrow 8x + 13 - 7x + 4z = 28 \\ x = 15 - 4z$$

$$20. \begin{cases} -2x + 2y - z = 0 \\ 3x - 4y + z = 1 \\ 5x - 8y + z = 4 \\ \begin{cases} -2x + 2y - z = 0 \\ x - 2y = 1 \\ 3x - 6y = 4 \end{cases} \\ \begin{cases} -2x + 2y - z = 0 \\ x - 2y = 1 \\ 0 = 1 \end{cases} \end{cases}$$

no solution.

$$22. \begin{cases} 3x + y - z = 10 \\ 8x - y - 6z = -3 \\ 5x - 2y - 5z = 1 \\ \begin{cases} 3x + y - z = 10 \\ 11x - 7z = 7 \\ 11x - 7z = 21 \end{cases} \end{cases}$$

no solutions.

$$34. (a) \begin{cases} -2 = 1 + A + B + C \\ 3 = 8 + 4A + 2B + C \\ 20 = 27 + 9A + 3B + C \end{cases}$$

$$(b) \begin{cases} -3 = A + B + C \\ -5 = 4A + 2B + C \\ -7 = 9A + 3B + C \end{cases}$$

$$\begin{cases} -3 = A + B + C \\ -2 = 3A + B \\ -4 = 8A + 2B \\ \begin{cases} -3 = A + B + C \rightarrow C = -1 \\ -2 = 3A + B \rightarrow B = -2 \\ 0 = 2A \rightarrow A = 0 \end{cases} \end{cases}$$

$$38. \begin{cases} 184 = a + b + c \\ 136 = 4a + 2b + c \\ 56 = 9a + 3b + c \\ \begin{cases} 184 = a + b + c \\ -48 = 3a + b \\ -128 = 8a + 2b \end{cases} \\ \begin{cases} 184 = a + b + c \\ -48 = 3a + b \\ -32 = 2a \rightarrow a = -16 \end{cases} \rightarrow b = 0 \end{cases}$$

$$0 = 16x^2 + 200 \\ \frac{25}{2} = x^2 \\ x = \pm \frac{5}{\sqrt{2}} \\ \text{so } t = \frac{5}{\sqrt{2}} \rightarrow c = 200$$

$$40. \begin{cases} \ln \alpha - \ln \beta - \ln \gamma = 2 \\ 3 \ln \alpha + 5 \ln \beta - 2 \ln \gamma = 1 \\ 2 \ln \alpha - 4 \ln \beta + \ln \gamma = 2 \\ \begin{cases} \ln \alpha - \ln \beta - \ln \gamma = 2 \\ \ln \alpha + 7 \ln \beta = -3 \\ 3 \ln \alpha - 5 \ln \beta = 4 \end{cases} \\ \begin{cases} \ln \alpha - \ln \beta - \ln \gamma = 2 \\ \ln \alpha + 7 \ln \beta = -3 \\ -2 \ln \beta = -3 \rightarrow \ln \beta = \frac{3}{2} \rightarrow \ln \beta = -\frac{1}{2} \quad \beta = e^{-\frac{1}{2}} \end{cases} \\ \begin{cases} \ln \alpha - \frac{1}{2} - \ln \gamma = 2 \\ \frac{1}{2} - (-\frac{1}{2}) - \ln \gamma = 2 \\ -1 - \ln \gamma = 1 \quad \gamma = e^{-1} \end{cases} \end{cases}$$

$(15 - 4z, -23 + 7z, z)$

$$y = \frac{13 - 7(15 - 4z)}{4} = -23 + 7z$$

6.3# 2, 4, 6, 8, 10, 12, 14, 16, 18, 24, 28, 32, 36

2. (a) 2 by 2

$$16. \begin{pmatrix} 3 & -3 & 1 & 4 \\ 6 & 7 & -3 & -7 \\ 1 & -2 & -2 & -3 \end{pmatrix}$$

(b) 4 by 1

$$\begin{pmatrix} 3 & -3 & 1 & 4 \\ 15 & -9 & 0 & 5 \\ 7 & -8 & 0 & 5 \end{pmatrix}$$

4. 1 by 4

$$x = \frac{1}{3} \quad -8y = 5 - \frac{7}{3} = \frac{8}{3}$$

6. $\begin{pmatrix} 5 & -1 & 1 \\ 0 & 4 & 2 \\ 3 & 1 & 1 \end{pmatrix}$

$$y = -\frac{1}{3}$$

$$1 + 1 + z = 4$$

$$z = 2$$

aug. $\begin{pmatrix} 5 & -1 & 1 & 0 \\ 0 & 4 & 2 & -1 \\ 3 & 1 & 1 & -1 \end{pmatrix}$

18. $\begin{pmatrix} 6 & 7 & -1 & -1 \\ -3 & 2 & 2 & 2 \\ 0 & 5 & 3 & 1 \end{pmatrix}$

8. $\begin{pmatrix} 8 & -8 & 0 \\ 1 & -1 & 1 \end{pmatrix}$

$\begin{pmatrix} -3 & 5 & 3 & 3 \\ 0 & 5 & 3 & 1 \end{pmatrix}$

aug $\begin{pmatrix} 8 & -8 & 0 & 5 \\ 1 & -1 & 1 & 1 \end{pmatrix}$

no solution

10. $\begin{pmatrix} 2 & -3 & 4 & 14 \\ 3 & -2 & 2 & 12 \\ 4 & 5 & -5 & 16 \end{pmatrix}$

24. $\begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 2-1 & 3-(-1) \\ -1-3 & 4-0 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ -4 & 4 \end{pmatrix}$$

$\begin{pmatrix} 2 & -3 & 4 & 14 \\ 3 & -2 & 2 & 12 \\ 1 & 7 & -7 & 4 \end{pmatrix}$

28. $\begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 2 + (-1) \cdot (-1) & 1 \cdot 3 + (-1) \cdot 4 \\ 3 \cdot 2 + 0 \cdot (-1) & 3 \cdot 3 + 0 \cdot 4 \end{pmatrix}$

$\begin{pmatrix} 0 & -17 & 18 & 6 \\ 0 & -23 & 23 & 0 \\ 1 & 7 & -7 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 3 & -1 \\ 6 & 9 \end{pmatrix}$$

$\begin{pmatrix} 0 & -17 & 18 & 6 \\ 0 & -23 & 23 & 0 \\ 1 & 7 & -7 & 4 \end{pmatrix}$

32. $\begin{pmatrix} 2 & 1 \\ 8 & -1 \\ 6 & 5 \end{pmatrix} + \begin{pmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 4 & -1 \\ 8 & 8 \end{pmatrix}$

$\begin{pmatrix} 0 & -17 & 18 & 6 \\ 0 & -23 & 23 & 0 \\ 1 & 7 & -7 & 4 \end{pmatrix}$

$z = 6, y = 6, x = 4$

36. $\begin{pmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 \cdot 5 + 2 \cdot (-4) + 3 \cdot 2 & -1 \cdot (-1) + 2 \cdot 0 + 3 \cdot 3 \\ 4 \cdot 5 + 0 \cdot (-4) + 5 \cdot 2 & 4 \cdot (-1) + 0 \cdot 0 + 5 \cdot 3 \end{pmatrix}$

12. $\begin{pmatrix} 5 & 1 & 10 & 23 \\ 4 & 2 & -10 & 76 \\ 3 & -4 & 0 & 18 \end{pmatrix}$

$\begin{pmatrix} 5 & 1 & 10 & 23 \\ 9 & 3 & 0 & 79 \\ 3 & -4 & 0 & 18 \end{pmatrix}$

$\begin{pmatrix} 5 & 1 & 10 & 23 \\ 0 & 15 & 0 & 45 \\ 3 & -4 & 0 & 18 \end{pmatrix}$

$$= \begin{pmatrix} -7 & 10 \\ 30 & 11 \end{pmatrix}$$

$y = 3 \quad x = 10 \quad z = -3$

14. $\begin{pmatrix} 2 & 3 & -4 & 7 \\ 1 & 0 & -1 & -7 \\ 0 & -5 & -2 & -7 \end{pmatrix}$

$\begin{pmatrix} 2 & 3 & -4 & 7 \\ 2 & 6 & -5 & -7 \\ 0 & -5 & -2 & -7 \end{pmatrix}$

$\begin{pmatrix} 2 & 3 & -4 & 7 \\ 0 & -5 & 6 & -10 \\ 0 & -14 & 10 & -28 \end{pmatrix}$

$\begin{pmatrix} 2 & 3 & -4 & 7 \\ 0 & -5 & 6 & -10 \\ 0 & -7 & 5 & -14 \end{pmatrix}$

$\begin{pmatrix} 2 & 3 & -4 & 7 \\ 0 & 2 & 1 & 4 \\ 0 & -7 & 5 & -14 \end{pmatrix}$

$\begin{pmatrix} 2 & 3 & -4 & 7 \\ 0 & 2 & 1 & 4 \\ 0 & -17 & 0 & -34 \end{pmatrix}$

$y = 2 \quad z = 0$

$2x + 6 = 7 \quad x = \frac{1}{2}$

6.6 # 4, 10, 14, 18, 20, 40, 43, 46, 47, 48, 49, 50, 51, 52, 57

4. $4y = 12 - 3x$

$y = 3 - \frac{3}{4}x$

$x^2 - (3 - \frac{3}{4}x) + 1 = 0$

$x^2 + \frac{3}{4}x - 2 = 0$

$4x^2 + 3x - 8 = 0$

$x = \frac{-3 \pm \sqrt{9 + 4(4)8}}{2(4)}$

$x = \frac{-3 \pm \sqrt{137}}{8}$

$(\frac{-3 + \sqrt{137}}{8}, 3 - \frac{3}{4}(\frac{-3 + \sqrt{137}}{8}))$

$(\frac{-3 - \sqrt{137}}{8}, 3 - \frac{3}{4}(\frac{-3 - \sqrt{137}}{8}))$

10. $x(4x) = 4$

$x^2 = 1 \quad x = \pm 1$

$(1, 4) \text{ or } (-1, -4)$

14. $(x-3)^2 + (-\sqrt{x})^2 = 4$

$x^2 - 6x + 9 + x = 4$

$x^2 - 5x + 5 = 0$

$x = \frac{5 \pm \sqrt{25 - 4(5)}}{2} = \frac{5 \pm \sqrt{5}}{2}$

$(\frac{5 + \sqrt{5}}{2}, -\sqrt{\frac{5 + \sqrt{5}}{2}}) \text{ or } (\frac{5 - \sqrt{5}}{2}, -\sqrt{\frac{5 - \sqrt{5}}{2}})$

18. $e^{4x} - e^{2x} - 6 = 0$

$(e^{2x} - 3)(e^{2x} + 2) = 0$

$e^{2x} = 3 \text{ or } e^{2x} = -2$

$2x = \ln 3$

$x = \frac{1}{2} \ln 3$

$y = e^{2 \ln 3} = 9$

20. $\log_2(x+1) + \log_2(x-3) = 5$

$(x+1)(x-3) = 32$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7 \text{ or } x = -5$

$y = \log_2(7+1) = \log_2 8 = 3$

$(7, 3)$

40. $y = \frac{r^2}{z} \quad x = \frac{q^2}{z}$

$\frac{r^2}{z} \frac{q^2}{z} = r^2 \quad \frac{r^2 q^2}{r^2} = z^2$

$z = \pm \frac{rq}{r} \quad z = \frac{rq}{r} \quad \text{then } x = \frac{q^2}{r} \quad y = \frac{r^2}{q}$

$(\frac{rq}{r}, \frac{r^2}{q}, \frac{rq}{r}) \text{ or } (-\frac{rq}{r}, -\frac{r^2}{q}, -\frac{rq}{r})$

43. (a) $3 = N_0 e^{2k} \quad 24 = N_0 e^{8k}$

$8 = e^{6k} \Rightarrow k = \frac{1}{6} \ln 8$

$N_0 = \frac{3}{e^{\ln(2)}} = \frac{3}{2}$

(b) $1 = N_0 e^{\frac{1}{2}k} \quad 10 = N_0 e^{4k}$

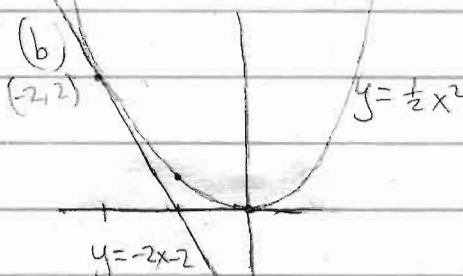
$10 = e^{\frac{7}{2}k} \quad k = \frac{2 \ln 10}{7}$

$N_0 = \frac{1}{e^{\frac{1}{2} \ln 10}} = \frac{1}{\sqrt{10}}$

46. (a) $-2x - 2 = \frac{1}{2}x^2$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0 \rightarrow x = -2 \quad y = 2$



47. $\frac{1}{2}xy = 180 \quad \sqrt{x^2 + y^2} = 41$

$y = \frac{360}{x} \quad x^2 + (\frac{360}{x})^2 = 1681$

$x^4 - 1681x^2 + 129600 = 0$

$z^2 - 1681z + 129600 = 0$

$z = \frac{1681 \pm \sqrt{1519}}{2} \quad z = 1600 \text{ or } z = 81$

$z = \pm 40 \quad z = \pm 9$

let $z = x^2$

$\left. \begin{array}{l} x = 40 \text{ or } x = 9 \\ y = 9 \end{array} \right\} \begin{array}{l} y = 40 \end{array}$

6.6 Continued.

$$48. \begin{cases} x+y=8 \rightarrow y=8-x \\ xy=-128 \\ x(8-x)=-128 \\ -x^2+8x=-128 \\ 0=x^2-8x-128 \\ = (x-16)(x+8) \\ \begin{cases} x=16 & \text{or} & x=-8 \\ y=-8 & & y=16 \end{cases} \end{cases}$$

$$49. \begin{cases} 2x+2y=46 \\ xy=60 \rightarrow y=\frac{60}{x} \\ 2x+2\left(\frac{60}{x}\right)=46 \\ 2x^2+120=46x \end{cases}$$

$$2x^2-46x+120=0$$

$$x^2-23x+60=0$$

$$(x-3)(x-20)=0$$

$$\begin{cases} x=3 & \text{or} & x=20 \\ y=20 & & y=3 \end{cases}$$

$$50. \frac{1}{2}xy=24$$

$$y=\frac{48}{x}$$

$$x+y+\sqrt{x^2+y^2}=24$$

$$\sqrt{x^2+y^2}=24-x-y$$

$$x^2+y^2=576-48x-48y+2xy+x^2+y^2$$

$$0=576-48x-48y+2xy$$

$$0=576-48x-48\left(\frac{48}{x}\right)+2x\left(\frac{48}{x}\right)$$

$$0=576-48x-\frac{2304}{x}+96$$

$$48x^2-672x+2304=0$$

$$x^2-14x+48=0$$

$$(x-6)(x-8)=0$$

$$\begin{cases} x=6 & \text{or} & x=8 \\ y=8 & & y=6 \end{cases}$$

$$51. y=\frac{2}{x}$$

$$x^2+\left(\frac{2}{x}\right)^2=5$$

$$x^4+4-5x^2=0$$

$$(x^2-4)(x^2-1)=0$$

$$\begin{cases} x=2 & \text{or} & x=-2 \\ y=1 & & y=-1 \end{cases} \quad \text{or} \quad \begin{cases} x=-1 & \text{or} & x=1 \\ y=2 & & y=-2 \end{cases}$$

$$52. x^2+2xy+y^2=9$$

$$(x+y)^2=9 \rightarrow x+y=\pm 3$$

$$x^2-2xy+y^2=1$$

$$(x-y)^2=1 \rightarrow x-y=\pm 1$$

$$\begin{cases} x+y=3 \\ x-y=1 \end{cases} \rightarrow \begin{cases} y=1 \\ x=2 \end{cases} \quad \text{or} \quad \begin{cases} x+y=3 \\ x-y=-1 \end{cases} \rightarrow \begin{cases} y=2 \\ x=1 \end{cases}$$

or

$$\begin{cases} x+y=-3 \\ x-y=1 \end{cases} \rightarrow \begin{cases} y=-2 \\ x=-1 \end{cases} \quad \text{or} \quad \begin{cases} x+y=-3 \\ x-y=-1 \end{cases} \rightarrow \begin{cases} y=-1 \\ x=-2 \end{cases}$$

$$57. \begin{cases} x^4=y^6 \rightarrow y=\pm x^{2/3} \\ \ln\left(\frac{x}{y}\right)=\frac{\ln x}{\ln y} \end{cases} \quad \left(y = -x^{2/3} \text{ not in domain} \right)$$

$$\ln\left(\frac{x}{x^{2/3}}\right)=\frac{\ln x}{\ln(x^{2/3})}$$

$$\ln(x^{1/3})=\log_{x^{2/3}} x \quad \left(= \log_{x^{2/3}} (x^{3/2}) \right) = \frac{3}{2}$$

$$\ln(x^{1/3})=\frac{3}{2}$$

$$x^{1/3}=e^{3/2}$$

$$x=e^{9/2}$$

$$y=e^3$$