

7.2 # 4, 6, 10, 18, 20, 46, 48, 50, 52, 54, 56, 58, 59

$$\begin{array}{r}
 4. \quad 3x + 7 \\
 x-1 \overline{) 3x^2 + 4x - 1} \\
 \underline{3x^2 - 3x} \\
 7x - 1 \\
 \underline{7x - 7} \\
 6
 \end{array}$$

$$3x^2 + 4x - 1 = (x-1)(3x+7) + 6$$

$$\begin{array}{r}
 6. \quad x^2 - 3x^2 + 3x - 1 \\
 x-1 \overline{) x^4 - 4x^3 + 6x^2 - 4x + 1} \\
 \underline{x^4 - x^3} \\
 -3x^3 + 6x^2 \\
 \underline{-3x^3 + 3x^2} \\
 3x^2 - 4x \\
 \underline{3x^2 - 3x} \\
 -x + 1 \\
 \underline{-x + 1} \\
 0
 \end{array}$$

$$x^4 - 4x^3 + 6x^2 - 4x + 1 = (x-1)(x^3 - 3x^2 + 3x - 1) + 0$$

$$\begin{array}{r}
 10. \quad x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32 \\
 x-2 \overline{) x^5 - 2x^4 + 4x^3 - 8x^2 + 16x + 64} \\
 \underline{x^5 - 2x^4} \\
 4x^4 + 4x^3 - 8x^2 \\
 \underline{4x^4 - 8x^3} \\
 8x^3 + 8x^2 - 16x \\
 \underline{8x^3 - 16x^2} \\
 16x^2 + 16x + 64 \\
 \underline{16x^2 - 32x} \\
 32x + 64 \\
 \underline{32x - 64} \\
 128
 \end{array}$$

$$x^6 + 64 = (x-2)(x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32) + 128$$

$$\begin{array}{r}
 18. \quad z \\
 z^2 + z + 1 \overline{) z^3 + z^2 + z + 1} \\
 \underline{z^3 + z^2 + z} \\
 1
 \end{array}$$

$$z^3 + z^2 + z + 1 = z(z^2 + z + 1) + 1$$

$$\begin{array}{r}
 20. \quad ax^2 + (b+ra)x + (a^2+br+rc) \\
 x-r \overline{) ax^2 + bx^2 + cx + d} \\
 \underline{ax^2 - rax} \\
 (b+ra)x^2 + cx \\
 \underline{(b+ra)x^2 - r(b+ra)x} \\
 (a^2+br+rc)x + d \\
 \underline{(a^2+br+rc)x - ar^2-br^2-rc} \\
 ar^3 + br^2 + cr + d
 \end{array}$$

$$ax^3 + bx^2 + cx + d =$$

$$(x-r)(ax^2 + (b+ra)x + (a^2+br+rc)) + ar^3 + br^2 + cr + d$$

$$46. \quad \frac{4x^3 + 6x^2 - 6x - 5}{2x-3} = \frac{2x^3 + 3x^2 - 3x - \frac{5}{2}}{x - \frac{3}{2}}$$

$$\begin{array}{r|rrrr}
 \frac{3}{2} & 2 & 3 & -3 & -\frac{5}{2} \\
 & & 3 & 9 & 9 \\
 \hline
 & 2 & 6 & 6 & \frac{13}{2}
 \end{array}$$

$$q(x) = 2x^2 + 6x + 6 \quad R = 2\left(\frac{13}{2}\right) = 13$$

$$48. \quad \frac{5x^3 - 3x^2 + 1}{3x+1} = \frac{\frac{5}{3}x^3 - 1x^2 + \frac{1}{3}}{x + \frac{1}{3}}$$

$$\begin{array}{r|rrrr}
 -\frac{1}{3} & \frac{5}{3} & -1 & 0 & \frac{1}{3} \\
 & -\frac{5}{9} & \frac{14}{27} & -\frac{1}{27} & \frac{14}{81} \\
 \hline
 & \frac{4}{9} & -\frac{14}{27} & \frac{14}{27} & \frac{13}{81}
 \end{array}$$

$$q(x) = \frac{5}{3}x^2 - \frac{14}{27}x + \frac{14}{27} \quad R(x) = 3\left(\frac{13}{81}\right) = \frac{13}{27}$$

$$\begin{array}{r|rrrr}
 50. a) & -3 & 1 & 0 & k & 6 \\
 & & -3 & 9 & -3k-27 & \\
 \hline
 & 1 & -3 & k+9 & -3k-21 & R = -3k-21
 \end{array}$$

$$b) -3k - 21 = 0 \rightarrow -3k = 21 \quad k = -7$$

$$\begin{array}{r|rrrr}
 52. & 3 & 1 & -2 & -4 & 13 \\
 & & 3 & 3 & -3 & \\
 \hline
 & 1 & 1 & 1 & 0 &
 \end{array}$$

$$(x-3)(x^2 + x - 1) = 0$$

$$x=3 \quad x^2 + x - 1 = 0 \quad x = \frac{-1 \pm \sqrt{1 - (4)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\begin{array}{r|rrrr}
 54. & 3c & 1 & -2 & 0 & -4 \\
 & & 3c & -9-6c & -27c+18 & \\
 \hline
 & 1 & -2+3c & -9-6c & 14-27c &
 \end{array}$$

$$q(x) = x^2 + (-2+3c)x - 9-6c \quad R = 14-27c$$

$$\begin{array}{r|rrrr}
 56. & -2c & 1 & -1 & 4 & -4 \\
 & & -2c & -4+2c & 4 & \\
 \hline
 & 1 & -1-2c & 2c & 0 &
 \end{array}$$

$$q(x) = x^2 + (-1-2c)x + 2c \quad R = 0$$

$$58. \quad f(4) = d(4)q(4) + R(4)$$

$$f(4) = 0 \cdot q(4) + 321$$

$$f(4) = 321$$

$$\begin{array}{r|rrrr}
 59. & a & 1 & 0 & 0 & 0 & -5a^4 & 4a^5 \\
 & & a & a^2 & a^3 & a^4 & -4a^5 & \\
 \hline
 & 1 & a & a^2 & a^3 & -4a^4 & 0 &
 \end{array}$$

$$R = 0$$

7.3 # 2, 12, 15, 16, 20, 22, 24, 28, 30, 34, 38, 46, 48, 50, 52, 56, 61

2. $12(\frac{5}{4})^2 - (\frac{5}{4}) - 20 = \frac{75}{2}$, not a root.

12. $f(\sqrt{2}) = 1 + 2\sqrt{2} + (\sqrt{2})^2 - (\sqrt{2})^5 = 1$, not a zero.

15. $x=1, x=2(mult 3), x=3$

16. $x=0, x=-5(mult 4)$

$$\begin{array}{r|rrrr} 20. & 4 & 2 & 0 & -1 & -4 \\ & & 8 & 32 & & 124 \\ \hline & 2 & 8 & 31 & & 120 \end{array}$$

so $f(4) = 120$

$$\begin{array}{r|rrrrrr} 22. & -2 & 1 & -1 & -1 & -1 & -1 \\ & & -2 & 6 & -10 & +27 & -42 \\ \hline & 1 & -3 & 5 & -11 & 21 & -43 \end{array}$$

so $f(-2) = -43$

$$\begin{array}{r|rrrrrrr} 24. & -3 & 1 & -7 & 0 & 5 & 0 & 0 & 0 & 1 \\ & & -3 & 30 & -90 & 255 & -765 & +2145 & -6885 & \\ \hline & 1 & -10 & 30 & -85 & 255 & -765 & 2145 & -6884 & \end{array}$$

$$\begin{array}{r|rr} 28. & -1 & 7 & 11 & 5 \\ & & -1 & -6 & -5 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

$x^2 + 6x + 5 = 0$ roots: $x = -1, -5$
 $(x+5)(x+1) = 0$

$$\begin{array}{r|rr} 30. & -8 & 1 & 8 & -3 & -24 \\ & & -8 & 0 & & 24 \\ \hline & 1 & 0 & -3 & & 0 \end{array}$$

$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$ or $x = 8$

$$\begin{array}{r|rrrr} 34. & 4 & 6 & -19 & -25 & 18 & 8 \\ & & 24 & 20 & -20 & & -8 \\ \hline & -\frac{1}{3} & 6 & 5 & -5 & -2 & 0 \\ & & & -2 & -1 & & 2 \\ \hline & & 6 & 3 & -6 & & 0 \end{array}$$

$6x^2 + 3x - 6 = 0$

$x = \frac{-3 \pm \sqrt{9 + 144}}{12} = \frac{-3 \pm 3\sqrt{17}}{12}$
 $= -\frac{1 \pm \sqrt{17}}{4}$

Roots: $4, -\frac{1}{3}, \frac{-1+\sqrt{17}}{4}, \frac{-1-\sqrt{17}}{4}$

$$\begin{array}{r|rrrr} 38. & 2 & 6 & 5 & -24 & -25 & -5 & 0 \\ & & 0 & 0 & 0 & 0 & & 0 \end{array} \quad x=0 \text{ is a root}$$

$$\begin{array}{r|rrrr} & -\frac{1}{3} & 6 & 5 & -24 & -25 & -5 & 0 \\ & & -2 & -1 & 10 & & & 5 \end{array} \quad x = -\frac{1}{3}$$

$$\begin{array}{r|rrrr} & \sqrt{5} & 6 & 3 & -30 & -15 & & 0 \\ & & 6\sqrt{5} & 30\sqrt{5} & 15 & & & \end{array} \quad x = \sqrt{5}$$

$$\begin{array}{r|rr} & 6 & 30\sqrt{5} & 3\sqrt{5} & & 0 \end{array}$$

$6x^2 + (3+6\sqrt{5})x + 3\sqrt{5}$
 $(6x+3)(x+\sqrt{5})$
 $x = -\frac{1}{2}, x = -\sqrt{5}$

Roots: $-\frac{1}{2}, -\frac{1}{3}, 0, -\sqrt{5}, \sqrt{5}$

46. (a) -89 (from table $g(-3) = -89$)

(b) -22.15625

(c) $t+2$

$$\begin{array}{r|rrrr} (d) & -2 & 1 & 2 & 1 & 2 & -1 & -2 \\ & & -2 & 0 & -2 & 0 & & 2 \\ \hline & 1 & 0 & 1 & 0 & -1 & & 0 \end{array}$$

$x^4 + x^2 - 1 = 0$
 $y^2 + y - 1 = 0$
 $y = \frac{-1 \pm \sqrt{5}}{2}$

$x^2 = \frac{-1+\sqrt{5}}{2}$ or $x^2 = \frac{-1-\sqrt{5}}{2}$

$x = -2$ or $\sqrt{\frac{-1+\sqrt{5}}{2}}, -\sqrt{\frac{-1+\sqrt{5}}{2}}, \sqrt{\frac{-1-\sqrt{5}}{2}}, -\sqrt{\frac{-1-\sqrt{5}}{2}}$

48. $(x-\frac{1}{2})(x-\frac{2}{3})(x+\frac{3}{4}) \cdot 2 \cdot 5 \cdot 4$
 $= (2x-1)(5x-2)(4x+3)$

50. $(x-4)^2(x+1)^2 \leftarrow$ can't be degree 3, no such eq.

52. $(x-\frac{1}{2})^2(2x^2-4x-1) \cdot 4$
 $= (2x-1)^2(2x^2-4x-1)$

56. (a) no (b) yes.

$$\begin{array}{r|rr} 61. & a & 1 & 0 & -12 & 16 \\ & & a & a^2 & a^2-12a & a^2-12a+16 \\ \hline & a & 1 & a & a^2-12 & a^2-12a+16 \\ & & & a & 2a^2 & \\ \hline & 1 & 2a & 3a^2-12 & & \end{array}$$

$3a^2 - 12 = 0$
 $a^2 = 4$
 $a = \pm 2$

$x^3 - 12x + 16 = (x+2a)(x-a)^2$
 $= (x+4)(x-2)^2$

$x = -4$ or $x = 2$

only $a=2$ is a root.

7.4 #2, 4, 8, 12, 16, 20, 22, 24, 26a, 34, 36, 40, 43a.

2. (a) yes (b) no (c) no (d) no

4. $y = x^2 - 2x - 290$
 $= x^2 - 2x + 1 - 291$
 $= (x-1)^2 - 291$
yes, has 2 real roots.

6. $x^5 + x^3 + x^2 + x + 1 = 0$

deg 5

must cross x-axis

\Rightarrow at least one real root.

12. $x(x^2 - 2x - 3)$

$\times (x-3)(x+1)$

16. $(x - i\sqrt{5})(x + i\sqrt{5})$

20. $x^2(x-4) = x^3 - 4x^2$

22. $(x-2-i)(x-2+i) = x^2 - 4x + 5$

24. $(x-5)^2(x-1)^3(x-1+i)(x-1-i)$

26a. $x(x-1)^2(x-3)$

34y = $a(x-\sqrt{2})(x+\sqrt{2})(x-1)(x+1)$

$y = a(x^2-2)(x^2-1)$

$-20 = a(4-2)(4-1)$

$-20 = a(2)(3) \rightarrow a = -\frac{10}{3}$

$-\frac{10}{3}(x^2-2)(x^2-1)$

36. $(x-1-i\sqrt{3})(x-1+i\sqrt{3})$

$x^2 - 2x + 4$

40. $(x-6+5i)(x-6-5i)$

$1445 = 5(289) = 5 \cdot 17^2$

$x^2 - 12x + 61$

43a. $a = 3$ $b = 76$

$x = \sqrt[3]{\frac{76}{2} + \sqrt{\frac{76^2}{4} + \frac{3^3}{27}}} - \sqrt[3]{\frac{-76}{2} + \sqrt{\frac{76^2}{4} + \frac{3^3}{27}}}$

$= \sqrt[3]{38 + \sqrt{1444+1}} - \sqrt[3]{-38 + \sqrt{1444+1}}$

$= \sqrt[3]{38+17\sqrt{5}} - \sqrt[3]{-38+17\sqrt{5}} = 4.$

6.1 4, 25, 30, 32a, 36, 43, 56, 57, 60

4. no

$$\begin{aligned} \text{43 cont } y &= \frac{ab(a-b)}{a^2-b^2} \\ &= \frac{ab}{a+b} \end{aligned}$$

$$25. \begin{cases} 3x - 4y = 48 \\ 2x - y = \frac{7}{10} \end{cases}$$

$$\begin{cases} -5x = 48 - 4\left(\frac{7}{10}\right) \\ 2x - y = \frac{7}{10} \end{cases}$$

$$\text{so } x = -\frac{226}{25}$$

$$y = \frac{939}{50}$$

$$56. \begin{cases} x + y = 64 \\ 2x + 5y = 20 \\ x + y = 64 \end{cases}$$

$$3y = -108$$

$$\begin{cases} x + y = 64 \\ y = -36 \end{cases}$$

$$x = 100$$

$$30. \begin{cases} 11 = a - b + 1 \\ 1 = 9a + 3b + 1 \end{cases}$$

$$\begin{cases} 10 = a - b \\ 0 = 9a + 2b \end{cases}$$

$$\begin{cases} 10 = a - b \\ 30 = 12a \end{cases}$$

$$a = \frac{5}{2} \quad b = \frac{15}{2}$$

$$57. \begin{cases} x + y = 14 \\ 2x = y + 1 \\ x + y = 14 \\ 2x - y = 1 \end{cases}$$

$$3x = 15$$

$$\begin{aligned} x &= 5 \\ y &= 9 \end{aligned}$$

is 59

$$32a. \begin{cases} 1 = 8a + b \\ -7 = -8a + b \end{cases}$$

$$-6 = 2b$$

$$\begin{aligned} b &= -3 \\ a &= \frac{1}{2} \end{aligned}$$

$$60. \begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

$$\begin{cases} adx + bdy = cd \\ adx + aey = af \end{cases}$$

$$(bd - ae)y = cd - af$$

$$y = \frac{cd - af}{bd - ae}$$

$$ax + b\left(\frac{cd - af}{bd - ae}\right) = c$$

$$x = \frac{c - b\left(\frac{cd - af}{bd - ae}\right)}{a}$$

36. 1st mix x 2nd mix y

$$\begin{cases} x + y = 50 \\ .15x + .2y = .16(50) \end{cases}$$

$$\begin{cases} x + y = 50 \\ 3x + 4y = 160 \end{cases}$$

$$\begin{cases} x + y = 50 \\ 3x + 4y = 160 \end{cases}$$

$$\begin{cases} x + y = 50 \\ 2x + y = 10 \end{cases}$$

$$\begin{aligned} x &= 40, y = 10 \end{aligned}$$

$$43. \begin{cases} \frac{x}{a} + \frac{y}{b} = 1 \\ \frac{x}{b} + \frac{y}{a} = 1 \end{cases}$$

$$\begin{cases} bx + ay = ab \\ ax + by = ab \end{cases}$$

$$\begin{cases} abx + a^2y = a^2b \\ abx + b^2y = ab^2 \end{cases}$$

$$(a^2 - b^2)y = a^2b - ab^2$$

$$(a^2 - b^2)y = a^2b - ab^2$$