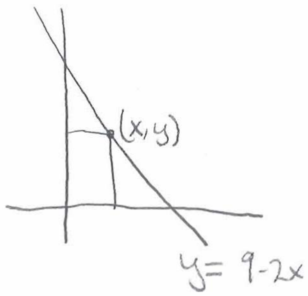
Max-Min Worksheet

1) The point (x, y) lies in the first quadrant on the graph of the line $y = 9 - 2x$. From this point, perpendicular lines are drawn to both the x-axis and the y-axis. What is the largest possible area for the rectangle thus formed?



$$\text{Area} = xy$$
$$= x(9-2x)$$
$$= 9x - 2x^2$$

when $x = \frac{-9}{(-2)2} = \frac{9}{4}$ this is max

So Max area $= 9\left(\frac{9}{4}\right) - 2\left(\frac{9}{4}\right)^2 = \frac{81}{4} - \frac{81}{8} = \frac{81}{8}$

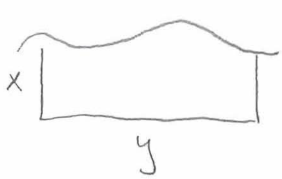
2) For what value(s) of a will the parabola $y = -x^2 + ax + 7$ have a maximum value of 10? Simplify as much as possible.

max when $x = \frac{-a}{2(-1)} = \frac{a}{2}$

max value $= -\left(\frac{a}{2}\right)^2 + a\left(\frac{a}{2}\right) + 7 = 10.$

$$-\frac{a^2}{4} + \frac{a^2}{2} + 7 = 10$$
$$\frac{a^2}{4} = 3$$
$$a^2 = 12 \rightarrow a = \pm\sqrt{12}$$

3) A farmer has \$10,000 to fence off a rectangular pasture along a river. The north side does not need to be fenced, as it is against the river. Because of winds, the south fence will cost \$10 per foot while the east and west fences will cost \$2 per foot. What dimensions maximize area?



Cost: $\$10,000 = \$2 \cdot x + \$10 \cdot y + \$2 \cdot x$

$10000 = 4x + 10y \rightarrow 10000 - 4x = 10y$

$y = 1000 - \frac{4}{10}x$

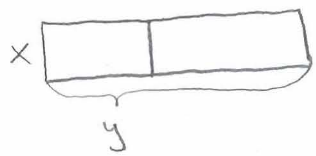
$$\text{Area} = xy$$
$$= x\left(1000 - \frac{4}{10}x\right)$$
$$= 1000x - \frac{4}{10}x^2$$

when $x = \frac{-1000}{2\left(-\frac{4}{10}\right)} = 1250$ this is max

$y = 1000 - \frac{4}{10}(1250) = 500$

Dimensions:
500 ft (on south)
by
1250 ft (on east and west)

4) Cecilia has 1000 dollars with which to build fencing for two adjacent rectangular corrals. The two corrals are to share a common fence on one side, as shown below. If the cost of the fence separating the two corrals from each other is 2 dollars per foot and the cost of the fences surrounding the corrals is 4 dollars per foot, what is the largest possible area of the fenced in region?



Cost: $\$1000 = \$4x + \$4y + \$4x + \$4y + \$2x$

$1000 = 10x + 8y \rightarrow y = \frac{1000 - 10x}{8}$

Area $= xy$

$= x\left(125 - \frac{5}{4}x\right)$

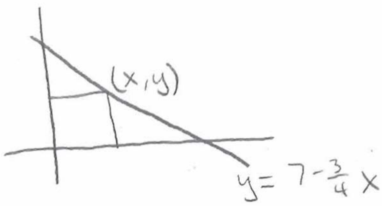
$= 125x - \frac{5}{4}x^2$

max when $x = \frac{-125}{2(-\frac{5}{4})} = 50$

$\rightarrow$ max Area $= 125(50) - \frac{5}{4}(50)^2$

$= 3125\ ft^2$

5) A point P lies in the first quadrant of the graph of the line $y = 7 - \frac{3}{4}x$. From the point P perpendicular lines are drawn to both the x-axis and y-axis, forming a rectangle. What is the largest possible area for this rectangle?



$A = xy$

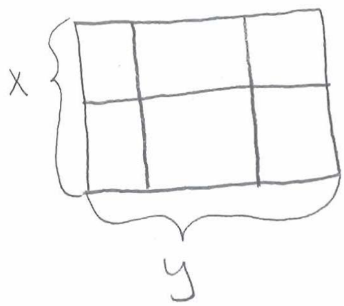
$A = x\left(7 - \frac{3}{4}x\right)$

$= 7x - \frac{3}{4}x^2$

max when $x = \frac{-7}{2(-\frac{3}{4})} = \frac{14}{3}$

max Area $= 7\left(\frac{14}{3}\right) - \frac{3}{4}\left(\frac{14}{3}\right)^2 = \frac{49}{3}$

6) A farmer has 1000ft of fencing to enclose 6 rectangular pastures as pictured. What dimensions maximize the total area of the 6 pastures?



Amount of fencing: $4x + 3y = 1000$

$y = \frac{1000}{3} - \frac{4}{3}x$

$A = xy$

$= x\left(\frac{1000}{3} - \frac{4}{3}x\right)$

$= \frac{1000}{3}x - \frac{4}{3}x^2$

max at $x = \frac{-\frac{1000}{3}}{2(-\frac{4}{3})^2} = 125$

$y = \frac{1000}{3} - \frac{4}{3}(125) = 166\frac{2}{3}$

Dimensions:

125ft by $166\frac{2}{3}$ ft