

## Review for Exam 2

Sections 3.1 and 3.2 : functions and their graphs

Domain

Range

Vertical line test

what it means to be a function

Key characteristics of graphs

The 6 standard graphs on the inside cover of the book

What does a point on the graph of a function represent?

Section 3.3 : Average rates of Change (difference quotients)

The average rate of change of  $f(x)$  on the interval  $[a, b]$  is  $\frac{f(b) - f(a)}{b - a}$  notice that this is also the slope between the points  $(a, f(a))$  and  $(b, f(b))$

problems from this section often require a bit of algebraic simplification, so I recommend looking over problems in the appendix for inspiration.

Section 3.4 : Techniques in Graphing

Graphing vertical and horizontal shifts, and reflections in the x-axis and y-axis.

Get the order correct.

Section 3.5 Combining Functions

Know definition of  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(fg)(x)$  and  $(\frac{f}{g})(x)$  and how to find domain

Most of this section deals with function composition i.e.  $(f \circ g)(x)$

know how to find domain of composition

this material also relates to section 3.4, thinking of shifts and reflections as compositions

Section 3.6 Inverse Functions

What is necessary for a function to have an inverse

know how to find the inverse

how does the graph of a function relate to the graph of its inverse?

domain of a function is the range of its inverse, and the range of a function is the domain of its inverse

Section 4.1 Linear Functions

models for depreciation, cost, position

what does slope represent, what does the y-intercept represent

all lines which are not horizontal or vertical have inverses

Section 4.2 Quadratic Functions

standard form  $y = a(x - h)^2 + k$  get by completing the square

has vertex  $(h, k)$ , opens up if  $a > 0$  (has min) and down if  $a < 0$  (has max)

axis of symmetry  $x = h$ , y-intercept always exists

the discriminant will tell you how many x-intercepts there are

$f(x) = ax^2 + bx + c$  has a maximum or minimum when  $x = \frac{-b}{2a}$

this max or min value is  $f(\frac{-b}{2a})$

#### Section 4.4 setting up equations

steps:

1. Read/absorb/draw a picture
2. pick quantities to assign variables to
3. set up equations

#### Section 4.5 Maximizing and Minimizing

4. your equation from 3. should be quadratic - find max/min as in section 4.2

#### Section 4.6 Polynomial Functions

Characteristics: eg domain  $(-\infty, \infty)$ , smooth, connected

shape of polynomials of even degree, of odd degree

the number of turning points is less than the degree

to graph a polynomial, find out where it is positive and negative with a numberline, find the y-intercept by plugging in  $x = 0$ , draw accordingly

near an x-intercept the graph will look like the power function with the exponent that produced that zero.

#### Section 4.7 Rational Functions

Know how to combine the graph of  $y = \frac{1}{x}$  with translations/reflections from section 3.4 in general:

zeros in numerator give x-intercepts, zeros from denominator give vertical asymptotes, y-intercepts come from plugging in  $x = 0$  (if 0 is in the domain)

Horizontal asymptotes take a few steps, the idea of these steps is to help figure out what the behaviour is for large values of  $|x|$ :

1. multiply out numerator and denominator, and factor the highest power of  $x$  out of each
2. cancel
3. see what happens for large  $x$

what happens is the following:

if the highest powers are equal, the HA will be the ratio of the coefficients of the highest powers

if the numerator has a higher power, there is no HA

if the denominator has a higher power, the HA is  $y = 0$

To finish the graph, make a positive/negative numberline, and draw accordingly. Some problems on the homework check where the graph intersects the HA, you may do this if the HA is  $y = c$  by solving the equation  $c = f(x)$

#### Section 5.1 Exponential functions

Characteristics of  $y = b^x$  for  $b > 0$  and  $b \neq 1$ :

Domain  $(-\infty, \infty)$ , Range  $(0, \infty)$

no x-intercept, y-intercept  $(0,1)$

no VA, HA  $y=0$

shape for  $b > 1$ , as opposed to shape for  $0 < b < 1$

combine this with section 3.4

these all have inverses, which we can get by looking at the reflection of the graph in the line  $y=x$   
properties of inverse

another type simplification problem shows up in this section  
solve equations with variables in the exponent by getting the same base on both sides,  $b^x = b^y$  and  
then solving the equation  $x = y$

Section 5.2 The exponential function  $e^x$

$e \approx 2.718$

$e$  is special because the tangent line at the y-intercept has slope 1

one other place  $e$  comes from is  $(1 + \frac{1}{x})^x$  gets close to  $e$  for large  $x$ , this is relevant to compound  
interest