

Dear Class,

Some questions on this homework are open-ended. You will be given more credit for more+better work. At the same time, you will not be penalized in any way if you avoid Wikipedia and choose instead to explore for yourself.

Noah

1. Compute  $\begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}$ .

2. Say what you think is the easiest possible matrix to compute with, and explain why. Your explanation should be as precise and clear as possible, i.e. not just “because its made of really easy numbers.”

3. What special properties does the matrix  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  have?

4. What special properties does the matrix  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  have?

5. We noticed in class that the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  has a very nice property, similar to the number 1. Namely, if we call this matrix “I” (for “identity,”) and any other matrix A, then

$$I^*A=A, \text{ and}$$

$$A^*I=A.$$

What special properties does the matrix  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$  have? Are there any numbers with this property?

6. We now know how to multiply matrices. Addition should be easier. Come up with a way to add matrices, keeping the following in consideration:

(a) What special properties would you like your operation of addition to have? What special properties does addition of numbers have? Some of these may involve the way addition and multiplication act when combined together.

(b) Does your definition of addition have these properties?

(c) We never really checked: does multiplication of matrices satisfy the nice properties that you already know for multiplication of numbers?

7. You all solved the following equation in class, and saw that it wasn't that easy.

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} X = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

Come up with three matrix algebra problems that you think are reasonably easy to solve. An unknown in these problems should be a whole matrix, not just one number in a matrix of four numbers. You are allowed to use your concept of matrix addition, just not for all three of your problems.

8. Erica said something in class that was once said by John von Neumann, a mathematical genius of the early 20<sup>th</sup> century: "In mathematics, you don't understand things. You just get used to them."

(a) Do you think this is correct?

(b) What do you think the difference is, between understanding something and getting used to it?