MATH 210 **PRACTICE** EXAM 2
Semester I, 2006-2007 Lecture 4

Name: _____ **Section:** _____

NO CALCULATORS, NOTES, BOOKS, ETC. ALLOWED.
EXPLAIN YOUR WORK.
ANSWERS WITHOUT EXPLANATION WILL RECEIVE 0 CREDIT.

Unless you are instructed otherwise, your answer should be computed completely (e.g., as a whole number, or a simple fraction, or a decimal).

A table of the areas under the standard normal curve is attached to this exam.

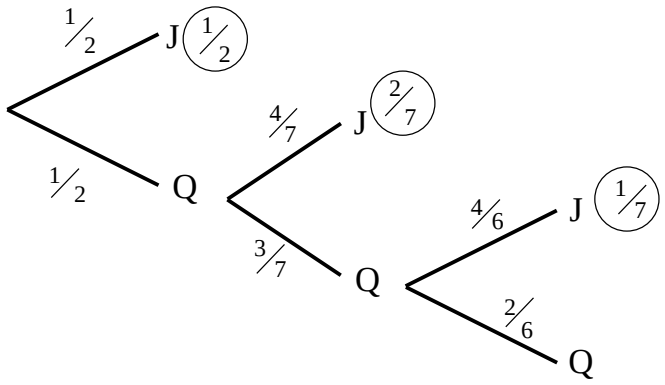
Number	MAX	Grade
1	10	
2	13	
3	13	
4	13	
5	13	
6	15	
7	13	
8	10	
SUM	100	

1. (10 points) An unfair coin has a probability of 0.9 of coming up heads. It is tossed 7 times. Assume that the results of the tosses are independent. Find the probability that your sequence of 7 tosses contains more heads than tails. You may leave your answer as an arithmetical expression, without evaluating it.

You need 4 or 5 or 6 or 7 heads, so the probability is

$$C(7,4)(.9)^4(.1)^3 + C(7,5)(.9)^5(.1)^2 + C(7,6)(.9)^6(.1)^1 + C(7,7)(.9)^7(.1)^0$$

2. (13 points) Consider the following experiment: A deck of 8 cards contains only 4 Jacks and 4 Queens. The deck is shuffled and the cards are dealt out one by one until a Jack appears. Let E be the event that a Jack appears as either the first, second, or third card dealt. Find $\Pr[E]$.



Probability $= 1/2 + 2/7 + 1/7 = 13/14$.

3. (13 points) A box contains five bills: two \$10 bills, two \$50 bills and one \$100 bill. From this box, you choose two bills at random. What is the expected value of the sum of the values of the two bills?

$n(S) = C(5, 2) = 10$, so counting up the sets which yield the various possible payoffs we get:

Payoff	prob	product
20	$\frac{1}{10}$	2
60	$\frac{4}{10}$	24
100	$\frac{1}{10}$	10
110	$\frac{2}{10}$	22
150	$\frac{2}{10}$	30
		$E(X) = \$88$

4. (13 points) A random variable X has the density function shown below. Find the expected value, variance, and standard deviation of X. Leave the result for $\sigma(X)$ in terms of a $\sqrt{\quad}$.

Value of X	Probability	Product	$X-\mu$	$(X-\mu)^2$	times Prob
-10	0.1	-1	-14	196	19.6
-5	0.2	-1	-9	81	16.2
5	0.4	2	1	1	.4
10	0.1	1	6	36	3.6
15	0.2	3	11	121	24.2
		$\mu = 4$			$Var = 64.0$

$\sigma(X) = \sqrt{64}$ OR $\sigma(X) = 8$ (either answer is OK here).

5. (13 points) Let X be a normal random variable with $\mu = E(X) = 12$ and $\sigma(X) = 2$. Find $\Pr[10 \leq X \leq 15]$.

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 12}{2}$$

so

$$\Pr[10 \leq X \leq 15] = \Pr\left[\frac{10 - 12}{2} \leq Z \leq \frac{15 - 12}{2}\right] =$$
$$\Pr[-1 \leq Z \leq 1.5] = 3413 + .4332 = .7745$$

6. (15 points) A bag with 450 fair dice is dumped onto a table. You get paid \$1 for each of the dice that comes up 2 or 6 (and nothing for the other dice). Use the normal approximation to the binomial to estimate the probabilities of the following happening:

a. You get paid between \$140 and \$170.

b. You get paid at least \$135.

$p = \frac{1}{3}$, $q = \frac{2}{3}$, $n = 450$, so $\mu = np = 150$ and $\sigma = \sqrt{npq} = \sqrt{100} = 10$.

QUICK WAY:

$$\Pr[140 \le X \le 170] = \Pr\left[\frac{-10}{10} \le Z \le \frac{20}{10}\right] = .3413 + .4772 = .8185$$

$$\Pr[135 \le X] = \Pr\left[\frac{-15}{10} \le Z\right] = 0.5 + \Pr\left[0 \le Z \le \frac{15}{10}\right] = .5 + .4332 = .9332$$

MORE ACCURATELY:

$$\Pr[139.5 \le X \le 170.5] = \Pr\left[\frac{-10.5}{10} \le Z \le \frac{20.5}{10}\right] = .3531 + .4798 = .8329$$

$$\Pr[134.5 \le X] = \Pr\left[\frac{-15.5}{10} \le Z\right] = 0.5 + \Pr\left[0 \le Z \le \frac{15.5}{10}\right] = .5 + .4394 = .9394$$

Either answer is OK.

7. (13 points) Let X be a random variable with probability density function f, where

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.2 & \text{if } 1 \le x < 4 \\ 0.4 & \text{if } 4 \le x < 5 \\ 0 & \text{if } 5 \le x \end{cases}$$

Find $\Pr[X \ge 2.5 \mid X \le 4.5]$ and $\Pr[X \le 4.5 \mid X \ge 2.5]$.

If E is the event "$X \ge 2.5$" and F is the event "$X \le 4.5$", then $E \cap F$ is the event "$2.5 \le X \le 4.5$". Looking at the graph and computing areas, we see that $\Pr[E] = .7$, $\Pr[F] = .8$, and $\Pr[E \cap F] = .5$. Then

$$\Pr[E \mid F] = \frac{\Pr[E \cap F]}{\Pr[F]} = \frac{5}{8} \qquad \Pr[F \mid E] = \frac{\Pr[E \cap F]}{\Pr[E]} = \frac{5}{7}$$

8. (10 points) Solve the system of equations:

$$4x + 5y = 14 \qquad\qquad 2x + 4y = 10$$

There is a unique solution here.

We have

$4x + 8y = 20$

$4x + 5y = 14$

Subtracting, we get $3y = 6$, or $y = 2$. Then, the equation on the left gives us $4x + 10 = 14$, so $x = 1$.

SCRAP PAPER

MORE SCRAP PAPER

Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990