MATH 210 – Section 4

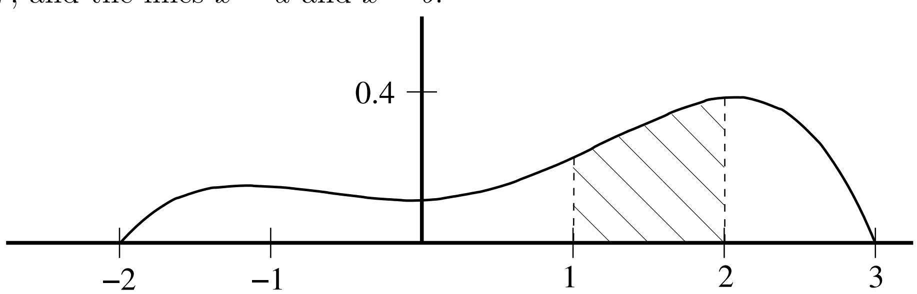
Finite Mathematics
Semester I, 2006-2007
CONTINUOUS RANDOM VARIABLES

This handout augments the material in Section 4.3 of the text.

Often, a random variable X has a continuum of possible values, rather than taking values from a finite set, and the probability that X is equal to any specific value is zero. In theory, X may take on any real number as a value, although in practice, we often have some bounded range into which we're 100% sure that X will fall.

For example, if X represents the weight of a random human in pounds, we don't have a finite list of possible values of X. It's fairly safe to say that $\Pr[0 \le X \le 10000] = 1$. It's also safe to say that the probability that you weigh a specific value, say, exactly 165.2319493214 pounds, is essentially zero; in symbols, $\Pr[X = a] = 0$ for each a.

Let $f(x)$ be a function such that $f(x) \ge 0$ for all x and the total area between the x axis and the graph of f is exactly 1. We say that a random variable X has *probability density function* $f(x)$ if $\Pr[X = a]$ is always 0, and, when $a < b$, $\Pr[a \le X \le b]$ is the area bounded by the x axis, the graph of f, and the lines $x = a$ and $x = b$.



In this picture, we define $f(x)$ to be $\frac{12}{625}(2+x)(3-x)(1+x^2)$ for $-2 \le x \le 3$ and 0 for other values. Then the area under the entire curve f is 1, and $\Pr[1 \le X \le 2]$ is exactly 0.32096 (you can see by eye that the shaded area is approximately 1/3 the total area). Unfortunately, to actually compute these values, you have to be able to compute the area under a curve given a formula for the curve. You learn that in calculus (take math 211).

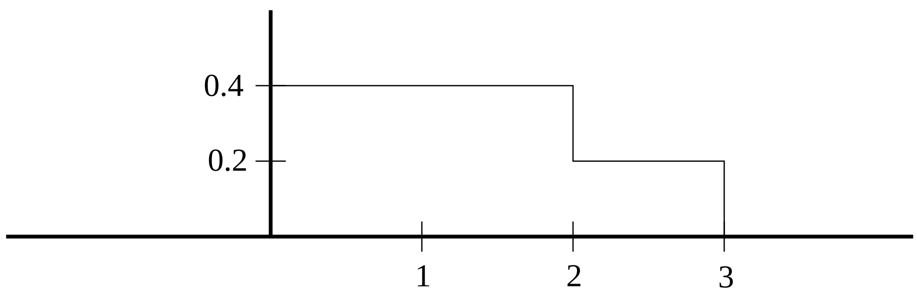
In this course, we'll only study examples where you can compute the area either by elementary geometry (see the example below), or by using the table in Appendix A of the text for the standard normal curve (see Section 4.3 for examples).

You all know how to compute the area of a rectangle, so you should be able to answer questions based on the definition:

$$\text{Let } f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.4 & \text{if } 0 \le x < 2 \\ 0.2 & \text{if } 2 \le x < 3 \\ 0 & \text{if } 3 \le x \end{cases}$$

Let X be a random variable with probability density function f. Then find:

1. $\Pr[1 \le X \le 2]$
2. $\Pr[X \le 1]$
3. $\Pr[X \ge 1.5]$
4. $\Pr[1 \le X \le 3]$
5. $\Pr[X \le 2 \mid X \ge 1]$
6. $\Pr[X \ge 1 \mid X \le 2]$
7. $\Pr[X \le 2.5 \mid X \ge 1.5]$
8. $\Pr[X \ge 1.5 \mid X \le 2.5]$



When you're given a problem like this, you should check that the total area under f is really equal to 1. Otherwise, the problem is meaningless.

Most practical problems result in a density function which is curvy, and we won't discuss that in this course, unless it's the normal distribution function.

A problem which would result in this particular f is: Imagine a 10 mile long straight road in the desert, with two bars along it – one at the 3 mile mark and one at the 7 mile mark. You're dropped at random along this road. Then the random variable X is your distance to the closest bar.

Answer to (3). This is the union of a 0.5×0.4 rectangle and a 1×0.2 rectangle, so the area is $(0.5 \cdot 0.4) + (1 \cdot 0.2) = 0.4$.

Answer to (6). $\Pr[A|B] = \Pr[A \cap B] / \Pr[B] = \Pr[1 \le X \le 2] \;/\; \Pr[X \le 2] = 0.4/0.8 = 1/2$.