1. (13 points) Sets $A, B, C$ are subsets of a universal set $U$, and they satisfy:

$$n(A) = 13 \quad n(B) = 15 \quad n(C) = 12 \quad n(A \cap B \cap C) = 1$$

$$n(A \cup B) = 25 \quad n(A \cup C) = 23 \quad n(B \cup C) = 24 \quad n(U) = 40$$

Find $n(A' \cap B' \cap C')$.

2. (12 points) Let $Pr$ be a probability measure on $S$ with $E, F \subset S$. Assume that $Pr[E \mid F] = \frac{1}{3}$, $Pr[F \mid E] = \frac{1}{2}$, and $Pr[E' \cap F'] = \frac{3}{5}$.

Find $Pr[E], Pr[F]$, and $Pr[E \cap F]$.

3. (13 points) Let $Pr$ be a probability measure on $S$ with $E, F, G \subset S$. Assume that they satisfy:

$$Pr[E] = 0.55 \quad Pr[F] = 0.5 \quad Pr[G'] = 0.45$$

$$Pr[E \cap F] = 0.3 \quad Pr[E \cap G'] = 0.25 \quad Pr[F \cap G'] = 0.3$$

Find $Pr[E' \cap F' \cap G]$.

4. (12 points) Two fair dice are rolled; so the sum of the two numbers is between 2 and 12. Let $E$ be the event that the sum of the numbers is 11 or 12. Let $F$ be the event that the sum of the numbers is 2, 3, or 11. Find $Pr[E \mid F]$ and $Pr[F \mid E]$.

5. (12 points) Consider the following experiment: You start with a deck of 5 cards, $\{\heartsuit 2, \heartsuit 3, \heartsuit 4, \heartsuit 5, \heartsuit 6\}$. Now, shuffle the deck and deal them out on the table, one at a time; STOP when the sum of the numbers is 6 or greater. Find the sample space of this experiment. You can either draw a tree here, or you can just list all the possible outcomes. Note that you never deal the same card twice.

6. (13 points) An unfair die has probabilities $w_1, w_2, w_3, w_4, w_5, w_6$ of coming up 1, 2, 3, 4, 5, 6, respectively. Assume that 1 and 6 are equally likely, 2 and 5 are equally likely, and 3 and 4 are equally likely, but 2 is twice as likely as 1, and 3 is twice as likely as 2. Find $w_1, w_2, w_3, w_4, w_5, w_6$. Your answers should be a simple fraction (of form $m/n$).

7. (13 points) Let $S$ be the set of ten numbers, $\{1, 2, 3, \ldots, 10\}$. How many subsets of $S$ contain the same number of even numbers as odd numbers? Your answer should be a whole number. Examples of such sets are:

$$\{2, 3\}, \quad \{2, 4, 8, 1, 7, 9\}, \quad \emptyset, \quad S$$

8. (12 points) Start with a stack of 50 bills: 10 each of $1$ bills, $5$ bills, $10$ bills, $20$ bills, and $50$ bills. You randomly choose 10 of them. What is the probability of choosing exactly 2 of each kind? Your answer should be of the form $\frac{\text{a product of whole numbers}}{\text{another product of whole numbers}}$; you don’t have to simplify it.