MATH 210 **PRACTICE** EXAM 1

Semester I, 2007-2008 *Lecture 1*

1. (13 points) Sets A, B, C are subsets of a universal set U, and they satisfy:

$$\begin{array}{llll} n(A) = 13 & n(B) = 15 & n(C) = 12 & n(A \cap B \cap C) = 1 \\ n(A \cup B) = 25 & n(A \cup C) = 23 & n(B \cup C) = 24 & n(U) = 40 \end{array}$$

Find $n(\, A' \cap B' \cap C' \,)$.

2. (12 points) Let Pr be a probability measure on S with $E, F \subset S$. Assume that $\Pr[E \mid F] = \frac{1}{3}$, $\Pr[F \mid E] = \frac{1}{2}$, and $\Pr[E' \cap F'] = \frac{3}{5}$.

Find $\Pr[E]$, $\Pr[F]$, and $\Pr[E \cap F]$.

3. (13 points) Let Pr be a probability measure on S with $E, F, G \subset S$. Assume that they satisfy:

$$\begin{array}{lll} \Pr[E] = 0.55 & \Pr[F] = 0.5 & \Pr[G'] = 0.45 \\ \Pr[E \cap F] = 0.3 & \Pr[E \cap G'] = 0.25 & \Pr[F \cap G'] = 0.3 \\ \Pr[E \cap F \cap G'] = 0.2 & & \end{array}$$

Find $\Pr[E' \cap F' \cap G]$.

4. (12 points) Two fair dice are rolled; so the sum of the two numbers is between 2 and 12. Let E be the event that the sum of the numbers is 11 or 12. Let F be the event that the sum of the numbers is 2, 3, or 11. Find $\Pr[E \mid F]$ and $\Pr[F \mid E]$.

5. (12 points) Consider the following experiment: You start with a deck of 5 cards, $\{\diamondsuit 2, \diamondsuit 3, \diamondsuit 4, \diamondsuit 5, \diamondsuit 6\}$. Now, shuffle the deck and deal them out on the table, one at a time; STOP when the sum of the numbers is 6 or greater. Find the sample space of this experiment. You can either draw a tree here, or you can just list all the possible outcomes. Note that you never deal the same card twice.

6. (13 points) An unfair die has probabilities $w_1, w_2, w_3, w_4, w_5, w_6$ of coming up $1, 2, 3, 4, 5, 6$, respectively. Assume that 1 and 6 are equally likely, 2 and 5 are equally likely, and 3 and 4 are equally likely, but 2 is twice as likely as 1, and 3 is twice as likely as 2. Find $w_1, w_2, w_3, w_4, w_5, w_6$. Your answers should be a simple fraction (of form $\frac{m}{n}$).

7. (13 points) Let S be the set of ten numbers, $\{1, 2, 3, \ldots, 10\}$. How many subsets of S contain the same number of even numbers as odd numbers? Your answer should be a whole number. Examples of such sets are:

$\{2, 3\}$, $\{2, 4, 8, 1, 7, 9\}$, $\emptyset$, S

8. (12 points) Start with a stack of 50 bills: 10 each of \$1 bills, \$5 bills, \$10 bills, \$20 bills, and \$50 bills. You randomly choose 10 of them. What is the probability of choosing exactly 2 of each kind? Your answer should be of the form $\frac{\text{a product of whole numbers}}{\text{another product of whole numbers}}$; you don't have to simplify it.