

MATH 210 **PRACTICE EXAM 3**
Semester I, 2007-2008 Lecture 1

Unless you are instructed otherwise, your answer should be computed completely (e.g., as a whole number, or a simple fraction, or a decimal).

1. (13 points) Let S be the set of points in the plane which satisfy the following inequalities:

$$3x + 2y \geq -6 \quad 3x - 2y \leq 6 \quad x - 4y \leq 5$$

Find all the corner points of S , and say whether S is bounded or unbounded.

2. (15 points) Find the maximum and minimum values of $x + y$ subject to the constraints:

$$2x - y \geq -1 \quad -x + 2y \geq -1 \quad 4x + 3y \leq 12$$

If the maximum and/or minimum doesn't exist, say so.

3. (15 points) Solve the system of equations:

$$\begin{aligned} 3x + 4y + 7z &= 12 \\ 2x + 4y + 6z &= 8 \\ 4x + 2y + 5z &= 6 \end{aligned}$$

There is a unique solution here.

4. (12 points) Solve the system of equations:

$$\begin{aligned} 4x + 11y + 6z - 2t &= 4 \\ 3x + 6y - 3z + 9t &= 9 \end{aligned}$$

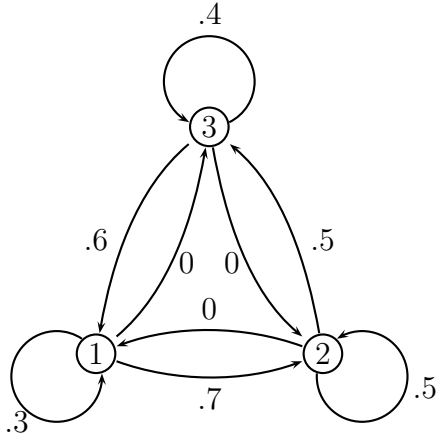
Here, two of the variables can be arbitrary, with the other two expressed in terms of them.

5. (10 points) Formulate the following as a linear optimization problem. Say explicitly what the variables stand for, what the constraints are, and what the objective function is. Don't solve the problem.

You're selling 25 pound sacks of mixed bird seed. Mix 1 contains 30% sunflower seeds, 45% peanuts, and 25% white millet. Mix 2 contains 50% sunflower seeds, 20% peanuts, and 30% white millet. Mix 3 contains 40% sunflower seeds, 35% peanuts, and 25% white millet. Mix 4 contains 40% sunflower seeds, 25% peanuts, and 35% white millet. You have on hand 1 ton of sunflower seeds, 2 tons of peanuts, and 3 tons of white millet. Your profits are \$3 for each sack of mix 1, \$2 for each sack of mix 2, \$5 for each sack of mix 3, and \$4 for each sack of mix 4. Assume that you know you will sell all the sacks you make up. How many sacks of each mix should you prepare to maximize your profits?

1 ton = 2000 pounds.

6. (10 points) A Markov chain has three states: State 1, State 2, State 3. Assume that it has the transition probabilities given by the diagram below and that it is known to be in State 1 on the initial observation. Find the probability that it is in each of the States 1,2,3 after two transitions.



7. (15 points) A Markov chain has three states: State 1, State 2, State 3. It has the transition matrix P shown below. Find the vector W of stable probabilities (so, $WP = W$). There is a unique solution here.

$$P = \begin{bmatrix} .6 & .3 & .1 \\ .1 & .6 & .3 \\ .1 & .1 & .8 \end{bmatrix}$$

8. (10 points) Say which of the following transition matrices is regular, and explain why; one is regular and one is not.

$$P = \begin{bmatrix} .12 & .11 & .77 & 0 \\ 0 & .77 & 0 & .23 \\ .11 & 0 & .12 & .77 \\ 0 & .25 & 0 & .75 \end{bmatrix} \quad Q = \begin{bmatrix} .23 & 0 & .77 & 0 \\ 0 & .29 & 0 & .71 \\ 0 & .57 & .43 & 0 \\ .39 & 0 & 0 & .61 \end{bmatrix}$$