

MATH 210 PRACTICE FINAL
Semester I, 2007-2008 Lecture 1

Unless you are instructed otherwise, your answer should be computed completely (e.g., as a whole number, or a simple fraction, or a decimal).

1. (7 points) All the diamonds are removed from a deck of cards, so it has 13 red cards and 26 black cards. Then, a five card hand is dealt at random from this deck. What is the probability that the hand contains more black cards than red cards? Here, you may leave your answer as an arithmetical expression, without evaluating it.

2. (9 points) Let \Pr be a probability measure on S with $E, F, G \subset S$. Assume that $\Pr[E'] = \frac{7}{13}$, $\Pr[F] = \frac{6}{13}$, $\Pr[G'] = \frac{7}{13}$, $\Pr[E' \cap F] = \frac{3}{13}$, $\Pr[E' \cap G'] = \frac{4}{13}$, $\Pr[F \cap G'] = \frac{4}{13}$, and $\Pr[E' \cap F \cap G'] = \frac{2}{13}$. Find each of $\Pr[E \cup F \cup G]$ and $\Pr[E' \cup F \cup G]$.

3. (7 points) Consider the following game: You roll a fair die once, and then get paid the *square* of the number you roll. So, if you roll a 2, you get \$4; if you roll a 5, you get \$25.

Find the expected value of your payoff.

4. (10 points) Consider the following game: I have a wad of six bills: three \$1 bills, two \$2 bills, and one \$5. Out of this wad, you choose two of them at random, and keep them (and not the other four).

Find the expected value, the variance, and the standard deviation of the amount of money you get to keep. You may leave the result for $\sigma(X)$ in terms of a $\sqrt{\quad}$.

5. (10 points) Solve the system of equations:

$$\begin{aligned}x + 3y - 13z &= 7 \\4x - 8y + 4z &= 4 \\x - 3y + 4z &= 6\end{aligned}$$

There is only one solution.

6. (10 points) A Markov chain has three states: State 1, State 2, State 3. It has the transition matrix P shown below. Find the vector W of stable probabilities (so, $WP = W$). There is a unique solution here.

$$P = \begin{bmatrix} .6 & .2 & .2 \\ .3 & .6 & .1 \\ .1 & .3 & .6 \end{bmatrix}$$

7. (8 points) You plant 1,000,000 corn seeds. Each seed has a germination rate of 90% (that is, it has a probability of .9 of sprouting). Use the normal approximation to the binomial to estimate the probabilities of the following happening:

- The number of seeds that sprout is between 900,000 and 900,450.
- The number of seeds that sprout is at least 900,030.

8. (8 points) Let X be a random variable with probability density function f , where

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.1 & \text{if } 0 \leq x < 6 \\ 0.4 & \text{if } 6 \leq x < 7 \\ 0 & \text{if } 7 \leq x \end{cases}$$

Find $\Pr[X \geq 3 \mid X \leq 5]$ and $\Pr[X \leq 5 \mid X \geq 3]$.

9. (7 points) A fair coin is tossed 8 times. Find the probability that your sequence of 8 tosses comes up heads exactly 5 times.

10. (7 points) Suppose that, starting now, you put \$50 into a bank account every month for the next twenty years. You start with a balance of zero. The bank pays 12% interest (annual rate), and the interest is compounded monthly. How much money will you have at the end of the twenty years? You may leave your answer as an arithmetical expression, without evaluating it.

11. (10 points) Find the maximum and minimum values of $x + y$ subject to the constraints:

$$-x + 3y \leq 6 \quad 2x + y \leq 4 \quad -2x + y \leq 4$$

If the maximum and/or minimum doesn't exist, say so.

12. (7 points) Suppose that you buy a ten year annuity with \$20,000. The annuity pays you monthly over the next ten years; after that, your money is completely used up. During the ten years, you continue to earn interest on the funds not yet paid out, at a 6% annual rate, compounded monthly. Find the monthly payments of this annuity. You may leave your answer as an arithmetical expression, without evaluating it.