MATH 210 **PRACTICE** FINAL

Semester I, 2006-2007 *Lecture 4*

1. A five card poker hand is dealt at random from a standard deck of 52 cards. What is the probability that the hand will contain at least one card from each of the four suits? Here, you may leave your answer as an arithmetical expression, without evaluating it.

2. Let Pr be a probability measure on S with $E, F, G \subset S$. Assume that $\Pr[E'] = 0.6$, $\Pr[F'] = 0.5$, $\Pr[G] = 0.55$, $\Pr[E' \cap F'] = 0.25$, $\Pr[E' \cap G] = 0.25$, $\Pr[F' \cap G] = 0.2$, and $\Pr[E' \cap F' \cap G] = 0.05$. Find each of $\Pr[E \cup F \cup G]$ and $\Pr[E' \cup F \cup G]$.

3. Consider the following game: Make up a 12–card deck using just the twos, threes, and fours; your deck has 6 red cards and 6 black cards. Then, you get dealt a hand of four cards from this deck. If you get four red cards, you get paid \$990. If you get exactly three red cards, you get paid \$495. If you get two or fewer red cards, you get paid nothing.

Find the expected value of your payoff.

4. A random variable X has the density function shown below. Find the expected value, variance, and standard deviation of X.

Value of X	Probability
-10	0.1
-4	0.1
-2	0.2
-1	0.1
1	0.1
2	0.2
4	0.1
10	0.1

5. Let X be a random variable with probability density function f, where

$$f(x) = \begin{cases} 0 & \text{if } x < -4 \\ 0.1 & \text{if } -4 \le x < 0 \\ 0.3 & \text{if } 0 \le x < 2 \\ 0 & \text{if } 2 \le x \end{cases}$$

Find $\Pr[X \ge -3 \mid X \le 1]$ and $\Pr[X \le 1 \mid X \ge -3]$.

6. A fair die is rolled 9 times. Assume that the results of the rolls are independent. Find the probability that your sequence of 9 rolls comes up 6 at least 7 times. Here, you may leave your answer as an arithmetical expression, without evaluating it.

7. An unfair coin has a probability of 0.4 of coming up heads and 0.6 of coming up tails. It is tossed 150 times. Use the normal approximation to the binomial to estimate the probabilities of the following happening:

a. The number of heads is between 54 and 66.

b. The number of heads is at least 48.

There will be a table of the areas under the standard normal curve at the end of this exam.

8. Solve the system of equations:

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & 7 \\ 5x & + & 5y & - & 10z & = & 5 \\ 2x & + & y & + & z & = & 6 \end{array}$$

There is only one solution.

9. A Markov chain has three states: State 1, State 2, State 3. It has the transition matrix P shown below. Find the vector W of stable probabilities (so, $WP = W$). There is a unique solution here.

$$P = \begin{bmatrix} .6 & .2 & .2 \\ .2 & .7 & .1 \\ .2 & .2 & .6 \end{bmatrix}$$

10. Find the maximum and minimum values of $x+y$ subject to the constraints:

$$\begin{array}{rr} x + 5y \leq 6 & 3x - y \leq 2 \\ x - 3y \leq 2 & 5x + y \geq -6 \end{array}$$

If the maximum and/or minimum doesn't exist, say so.

11. Find the monthly payments that are necessary to amortize a loan of \$10,000 over 5 years with interest at 5% annual rate. Here, you may leave your answer as an arithmetical expression, without evaluating it.

12. You need to have some money ten years from now, so you put money in a sinking fund, depositing monthly into an account which pays 6% interest (annual rate), compounded monthly. Each month you deposit \$100. How much money will you have at the end of the ten years? Here, you may leave your answer as an arithmetical expression, without evaluating it.