Solution of the practice exam

1. Draw a Venn diagram, then carry out each part of the diagram, starting from the center, \( A \cap B \cap C \). The answer is 48. \( n(A \cup B \cup C) = 52 \).

2. \( p(F) = 1/3, p(E \cup F) = 2/3, p(E|F) = \frac{p(E \cap F)}{p(F)} = 1/2 \). Then
   \[ p(E \cap F) = 1/2p(F) = 1/6. \]
Recall the identity
\[ p(E \cap F) + p(E \cup F) = p(E) + p(F), \]
it gives us
\[ p(E) = p(E \cap F) + p(E \cup F) - p(F) = 2/3 + 1/6 - 1/3 = 1/2. \]
Question: what if \( p(E|F) = 1/2 \) is replaced by \( p(F|E) = 1/2 \)?

3. Recall identity
   \[ p(A \cup B) + p(A \cap B) = p(A) + p(B). \]
It is easy to figure out that
\[ p(E \cap F) = p(E) + p(F) - p(E \cup F) = 0.15 \]
\[ p(E \cap G) = p(E) + p(G) - p(E \cup G) = 0.1 \]
\[ p(F \cap G) = p(F) + p(G) - p(F \cup G) = 0.1 \]
Then we can use Venn diagram to carry out each part as we did in no. 1 to get \( p(E \cup F \cup G) \). There is another way to approach the problem.
\[ p(E \cup F \cup G) = p(E) + p(F) + p(G) - p(E \cap F) - p(F \cap G) - p(E \cap G) + p(E \cap F \cap G) \]
\[ = 0.3 + 0.45 + 0.5 - 0.15 - 0.1 - 0.1 + 0.05 = 0.95. \]
Convince yourself by using Venn diagram that the above identity is right (for any sets).

4. Recall that
\[ p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{n(E \cap F)}{n(F)} \]
\( F = \{ \text{contains exactly one Queen in 3 cards} \} \)

\[ n(F) = C(4, 1)C(8, 2) \]

\( E \cap F = \{ \text{contains exactly one Queen, one Jack (and one King, why?)} \} \)

\[ n(E \cap F) = C(4, 1)C(4, 1)C(4, 1) \]

\[ p(E|F) = \frac{C(4, 1)C(4, 1)C(4, 1)}{C(4, 1)C(8, 2)} = \frac{4}{7}. \]

5. I just carry out the outcomes of sample space

\{3, 4, 5, 6, 21, 22, 23, 24, 25, 26, 111, 112, 113, 114, 115, 116\}

6. red apple—\( w_1 \), green apple—\( w_2 \), banana—\( w_3 \), \( w_1 = w_2 = 2w_3 \)

the probability of grabbing one apple is \( w_1 + w_2 \).

We know that \( w_1 + w_2 + w_3 = 1 \). it gives us that \( w_3 = 1/5 \), so \( w_1 + w_2 = 4/5 \).

7. subsets have 7 or more elements:

- exactly 7
  \[ C(10, 7) = C(10, 3) = 120 \]
- 8
  \[ C(10, 8) = C(10, 2) = 45 \]
- 9
  \[ C(10, 9) = C(10, 1) = 10 \]
- 10
  \[ C(10, 10) = 1 \]

So we have \( 120 + 45 + 10 + 1 = 176 \) subsets.

8. hands of random 9 cards: \( A = C(52, 9) \).

- hands of exactly 3 clubs, 2 diamonds, 2 spades (so exactly 2 hearts.):
  \[ a = C(13, 3)C(13, 2)C(13, 2)C(13, 2). \] The probability is \( a/A \) (carry out the exact answer as instruction.)