Solution of the practice exam

1. Draw a Venn diagram, then carry out each part of the diagram, starting from the center, $A \cap B \cap C$. The answer is 48. $n(A \cup B \cup C) = 52$.

2. $p(F) = 1/3, p(E \cup F) = 2/3, p(E|F) = \frac{p(E \cap F)}{p(F)} = 1/2$. Then

$$p(E \cap F) = 1/2p(F) = 1/6.$$

Recall the identity

$$p(E \cap F) + p(E \cup F) = p(E) + p(F),$$

it gives us

$$p(E) = p(E \cap F) + p(E \cup F) - p(F) = 2/3 + 1/6 - 1/3 = 1/2.$$

Question: what if $p(E|F) = 1/2$ is replaced by $p(F|E) = 1/2$?

3. Recall identity

$$p(A \cup B) + p(A \cap B) = p(A) + p(B).$$

It is easy to figure out that

$$p(E \cap F) = p(E) + p(F) - p(E \cup F) = 0.15$$

$$p(E \cap G) = p(E) + p(G) - p(E \cup G) = 0.1$$

$$p(F \cap G) = p(F) + p(G) - p(F \cup G) = 0.1$$

Then we can use Venn diagram to carry out each part as we did in no. 1 to get $p(E \cup F \cup G)$. There is another way to approach the problem.

$$\begin{aligned} p(E \cup F \cup G) &= p(E) + p(F) + p(G) - p(E \cap F) - p(F \cap G) - p(E \cap G) + p(E \cap F \cap G) \\ &= 0.3 + 0.45 + 0.5 - 0.15 - 0.1 - 0.1 + 0.05 = 0.95. \end{aligned}$$

Convince yourself by using Venn diagram that the above identity is right (for any sets).

4. Recall that

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{n(E \cap F)}{n(F)}$$

$F = \{$contains exactly one Queen in 3 cards$\}$

$n(F) = C(4,1)C(8,2)$

$E \cap F = \{$contains exactly one Queen, one Jack (and so one King, why?)$\}$

$n(E \cap F) = C(4,1)C(4,1)C(4,1)$

$$p(E|F) = \frac{C(4,1)C(4,1)C(4,1)}{C(4,1)C(8,2)} = 4/7.$$

5.I just carry out the outcomes of sample space

$$\{3, 4, 5, 6, 21, 22, 23, 24, 25, 26, 111, 112, 113, 114, 115, 116\}$$

6. red apple—w_1, green apple—-w_2, banana—-w_3, $w_1 = w_2 = 2w_3$

the probability of grabbing one apple is $w_1 + w_2$.

We know that $w_1 + w_2 + w_3 = 1$. it gives us that $w_3 = 1/5$, so $w_1 + w_2$=4/5.

7. subsets have 7 or more elements:
exactly

$$\begin{aligned} 7 &: C(10,7) = C(10,3) = 120 \\ 8 &: C(10,8) = C(10,2) = 45 \\ 9 &: C(10,9) = C(10,1) = 10 \\ 10 &: C(10,10) = 1 \end{aligned}$$

So we have $120 + 45 + 10 + 1 = 176$ subsets.

8. hands of random 9 cards: $A = C(52,9)$.

hands of exactly 3 clubs, 2diamonds, 2 spades (so exactly 2 hearts.):
$a = C(13,3)C(13,2)C(13,2)C(13,2)$. The probability is a/A (carry out the exact answer as instruction.)