

Name: \_\_\_\_\_

Math 311

Midterm Examination

March 15, 2005

**Part I.** Do all problems in the space provided. Clearly define all random variables and other notation that you use, and clearly specify what you are calculating at each step in a calculation. Justify all answers. Clearly mark all answers.

1. (8 pts) Three roommates take a test. Let

$$A = \{\text{Lou passes the test}\}$$

$$B = \{\text{Mary passes the test}\}$$

$$C = \{\text{Bob passes the test}\}$$

Using unions, intersections, complements, etc., express each of the following events in terms of  $A$ ,  $B$ , and  $C$ .

(a)  $D = \{\text{Only Bob passes the test}\}$

(b)  $E = \{\text{Exactly one student passes the test}\}$

2. (12 pts) Let  $P(A) = .2$  and  $P(B) = .5$ .

(a) If  $A$  and  $B$  are disjoint, what is  $P(A|B)$ ?

(b) If  $A$  and  $B$  are independent, what is  $P(A|B)$ ?

(c) If  $A \subset B$ , what is  $P(A|B)$ ?

3. (10 pts) Let  $X$  have probability density function

$$f_X(x) = \begin{cases} \frac{2}{9}x(3-x) & 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Calculate the cdf for  $X$ ?

(b) Calculate  $P\{1 < X < 2\}$ ?

4. (10 pts) Suppose  $Y$  has a discrete distribution with pmf given by

$y$	-1	0	1	2
$p_Y(y)$	1/2	1/4	1/8	1/8

(a) What is  $\mathcal{R}(Y)$ ?

(b) Calculate  $P\{Y > 0\}$ ?

(c) Calculate  $E[Y^2 - 1]$ ?

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1. \_\_\_\_\_ 4. \_\_\_\_\_  
2. \_\_\_\_\_ 5. \_\_\_\_\_  
3. \_\_\_\_\_ Total \_\_\_\_\_

**Part II.** Each of the following problems will be graded on the basis of 15 points. Your score on this part of the exam will be the sum of your 3 highest scores plus one half of the points you receive on the other two problems.

Clearly define all random variables and other notation that you use. Clearly mark your answers.

1. The probability that a newly manufactured radio is defective is .05. If an industrial engineer randomly checks 10 radios, what is the probability of finding:

(a) Exactly 2 defective radios.

(b) No more than 2 defective.

(c) At least 3 defective.

2. The instructor in a probability course has a fair die and a fair coin and performs the following experiment: First the die is rolled and if the number on the die is  $k$ , the instructor then flips the coin  $k$  times.

(a) What is the probability that all the flips are heads?

(b) If all the flips are heads, what is the probability that the number on the die was 1?

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3. A bucket contains six balls numbered 1 through 6. Three balls are drawn at random. Let  $X$  be the maximum of the three numbers and  $Y$  the minimum.

(a) Calculate the joint probability mass function for  $X$  and  $Y$ . Display the answer as a table.

(b) Calculate  $P\{Y = 2|X = 4\}$ .

4. Suppose that  $U$  and  $V$  are independent random variables with density functions

$$f_U(u) = \begin{cases} 2u & 0 \leq u \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad f_V(v) = \begin{cases} 2v & 0 \leq v \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate  $P\{U < V\}$ .

(b) Let  $Y = U + V$ . Calculate the pdf  $f_Y$  for  $Y$ .

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5. If an ice cream vendor spends  $c$  dollars per day on advertising, then the number of gallons of ice cream sold each day is uniformly distributed on the interval  $[0, \frac{20c}{1+c}]$ . Suppose each gallon of ice cream costs the vendor \$3 and the vendor sells the ice cream for \$10 per gallon. If the vendor's only income is from selling the ice cream and only expenses are for the ice cream and the advertising, how much should the vendor spend each day on advertising in order to maximize the expected daily profit.