1. Find the distance between the point $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

Solution:

\[d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]
\[= \sqrt{(6 - (-4))^2 + (2 - (-3))^2}\]
\[= \sqrt{10^2 + 5^2}\]
\[= \sqrt{125}\]
\[= 5\sqrt{5}\]

2. Find the midpoint of the line segment joining $P_1 = (2, -3)$ and $P_2 = (4, 2)$.

Solution:

\[M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]
\[= \left(\frac{2 + 4}{2}, \frac{-3 + 2}{2}\right)\]
\[= \left(3, -\frac{1}{2}\right)\]

3. Find all points on the y-axis that are 6 units from the point (4, -3).

Solution: Let $P_1 = (0, y)$ and $P_2 = (4, -3)$. We want $d(P_1, P_2) = 6$. So
6 = \sqrt{(4-0)^2 + (-3-y)^2} \\
36 = 16 + (-3-y)^2 \\
20 = 9 + 6y + y^2 \\
0 = y^2 + 6y - 11 \\
y = \frac{-6 \pm \sqrt{36 - 4(-11)}}{2} \\
= \frac{-6 \pm \sqrt{80}}{2} \\
= \frac{-6 \pm 4\sqrt{5}}{2} \\
= -3 \pm 2\sqrt{5} \\

Answer: (0, -3 + 2\sqrt{5}) and (0, -3 - 2\sqrt{5})

4. Find the intercepts of the equation \(5x + 2y = 10\). You might want to plot the points to illustrate the intercepts.

**Solution:** 1) the y-intercept is found when \(x = 0\). So, \(5(0) + 2y = 10 \implies y = 5\). Answer: (0, 5) 
2) the x-intercept is found when \(y = 0\). So, \(5(x) + 2(0) = 10 \implies x = 2\). Answer: (2, 0)

5. For the following points, find the point that is symmetric to it with respect to (a) the x-axis; (b) the y-axis; (c) the origin. You might want to plot the points to illustrate the symmetry.

(a) \((4, -2)\)

**Solution:** For (a) symmetry with respect to the x-axis, we replace \(y\) by \(-y\), we get \((4, 2)\)
For (b) symmetry with respect to the y-axis, we replace \(x\) by \(-x\), we get \((-4, -2)\)
For (c) symmetry with respect to the origin, we replace $x$ by $-x$ and $y$ by $-y$, we get $(-4, 2)$.

(b) $(4, 0)$

**Solution:**
For (a) symmetry with respect to the x-axis, we replace $y$ by $-y$, we get $(4, 0)$.
For (b) symmetry with respect to the y-axis, we replace $x$ by $-x$, we get $(-4, 0)$.
For (c) symmetry with respect to the origin, we replace $x$ by $-x$ and $y$ by $-y$, we get $(-4, 0)$.

6. Test the equation $x^2 - y - 4 = 0$ for symmetry.

**Solution:**
To test for symmetry with the x-axis, we replace $y$ by $-y$.
We get $x^2 - (-y) - 4 = 0 \Rightarrow x^2 + y - 4 = 0$.
Since $x^2 - y - 4 = 0$ is not equivalent to $x^2 + y - 4 = 0$, the graph of the equation is not symmetric with respect to the x-axis.

To test for symmetry with the y-axis, we replace $x$ by $-x$.
We get $(-x)^2 - y - 4 = 0 \Rightarrow x^2 - y - 4 = 0$.
Since $(-x)^2 - y - 4 = 0$ is equivalent to $x^2 - y - 4 = 0$, the graph of the equation is symmetric.
with respect to the y-axis.

To test for symmetry with the origin, we replace $x$ by $-x$ and $y$ by $-y$.
We get $(-x)^2 - (-y) - 4 = 0 \implies x^2 + y - 4 = 0$.
Since $x^2 - y - 4 = 0$ is not equivalent to $x^2 + y - 4 = 0$, the graph of the equation is not symmetric
with respect to the origin.

7. For the following, find an equation for the line with the given properties.

(a) Containing the points $(-3, 4)$ and $(2, 5)$.

**Solution:**

\[
\begin{align*}
m &= \frac{5 - 4}{2 - (-3)} = \frac{1}{5} \\
y - 5 &= \frac{1}{5}(x - 2) \text{ so the equation is } 5y - x = 23 \text{ or } y = \frac{1}{5}x + \frac{23}{5}
\end{align*}
\]

(b) x-intercept of $(-4, 0)$ and y-intercept of $(0, 4)$.

**Solution:**

\[
\begin{align*}
m &= \frac{4 - 0}{0 - (-4)} = 1 \text{ and } b = 4 \text{ so the equation is } y = x + 4 \text{ or } y = x + 4
\end{align*}
\]

(c) Vertical; containing the point $(3, 8)$.

**Solution:** The slope is undefined so the equation is $x = 3$

(d) Parallel to the line $x - 2y = -5$; containing the point $(0, 0)$.

**Solution:**

\[
\begin{align*}
x - 2y &= -5 \implies y = \frac{1}{2}x + \frac{5}{2} \text{ so } m = \frac{1}{2} \\
\text{Here } b = 0 \text{ so the equation is } y = \frac{1}{2}x \text{ or } x - 2y = 0
\end{align*}
\]
(e) Perpendicular to the line $x - 2y = -5$; containing the point $(0, 4)$.

**Solution:** We found in 7d) that $m = \frac{1}{2}$ so the slope of the perpendicular line has a slope of $m = -2$.
Here $b = 4$ so the equation is $y = -2x + 4$ or $2x + y = 4$.

8. Find the standard form of the equation of each circle.

(a) $r = 5$; $(h, k) = (4, -3)$

**Solution:** The standard form is $(x-h)^2 + (y-k)^2 = r^2$ so the equation is $(x-4)^2 + (y+3)^2 = 25$.

(b) With endpoints of a diameter at $(4, 3)$ and $(0, 1)$.

**Solution:** The center is the midpoint of the line segment joining $P_1 = (4, 3)$ and $P_2 = (0, 1)$.

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4 + 0}{2}, \frac{3 + 1}{2} \right) = (2, 2)
\]

The length of the radius is the distance between the center $(2, 2)$ and one of the endpoints of the diameter, let’s take $(0, 1)$.

\[
d = \sqrt{(2 - 1)^2 + (2 - 0)^2} = \sqrt{5}
\]

So the equation is $(x - 2)^2 + (y - 2)^2 = 5$.

(c) $x^2 + y^2 - 6x + 2y + 9 = 0$

**Solution:**

\[
\begin{align*}
x^2 + y^2 - 6x + 2y + 9 &= 0 \\
x^2 - 6x + y^2 + 2y &= -9 \\
x^2 - 6x + 9 + y^2 + 2y + 1 &= -9 + 9 + 1 \\
(x-3)^2 + (y+1)^2 &= 1
\end{align*}
\]

(d) $2x^2 + 2y^2 + 8x + 7 = 0$
Solution:

\[
\begin{align*}
2x^2 + 2y^2 + 8x + 7 &= 0 \\
2x^2 + 8x + 2y^2 &= -7 \\
2(x^2 + 4x) + 2y^2 &= -7 \\
2(x^2 + 4x + 4) + 2y^2 &= -7 + 8 \\
2(x + 2)^2 + 2y^2 &= 1 \\
(x + 2)^2 + y^2 &= \frac{1}{2}
\end{align*}
\]