COMPLETION OF NUMBERINGS

SERIKZHAN A. BADAEV, SERGEY S. GONCHAROV, AND ANDREA SORBI

Complete numberings play an important role in computability theory: they have an effective fixed point property, and their degrees are not splittable, while the completion operator gives us a regular way to construct a complete numbering α_a for any numbering $\alpha : \omega \longrightarrow \mathcal{A}$ and any special element $a \in \mathcal{A}$. We continue the study of the completion operator, started in [1].

As observed by Ershov, [2], $\alpha \leq \alpha_a$, and $\alpha_a \leq \beta$ for every numbering $\beta \geq \alpha$ which is complete with respect to a. In particular, $\alpha_a \equiv \alpha$ if and only if α is complete with respect to a.

We prove that no minimal numbering can be complete and, for every Friedberg numbering α, the interval $(\deg(\alpha), \deg(\alpha_a))$ consists of incomplete numberings (relative to any element of $\mathcal{A}$).

We show that, for some numbering α of a two-element set $\mathcal{A}$, the segment $[\deg(\alpha), \deg(\alpha_a)]$ is isomorphic to the upper semilattice of c.e. **m**-degrees. This is a partial solution to an open problem posed in [1].

REFERENCES

[1] S. Badaev, S. Goncharov, and A. Sorbi, *Completeness and universality of arithmetical numberings*, ***Computability and models*** (S. B. Cooper and S. S. Goncharov, editors), Kluwer / Plenum Publishers, New York, 2003, pp. 11–44.

[2] Yu. L. Ershov, ***Theory of numberings***, Nauka, Moscow, 1977.

KAZAKH NATIONAL UNIVERSITY
71 AL-FARABI AVE.
ALMATY, 050038, KAZAKHSTAN
E-mail: badaev@kazsu.kz

SOBOLEV INSTITUTE OF MATHEMATICS
4 ACAD. KOPTYUG
NOVOSIBIRSK, 480090 RUSSIA
E-mail: s.goncharov@math.nsc.ru

UNIVERSITY OF SIENA
44 PIAN DEI MANTELLINI
SIENA, 53100 ITALY
E-mail: sorbi@unisi.it