STRONG JUMP-TRACEABILITY I : THE COMPUTABLY ENUMERABLE CASE

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This talk will discuss a project which determines the relation between Turing degrees which are *super jump traceable* and those which are K*-trivial*. By work of Nies and others the K-trivals are very robust class of degrees; for example, see Nies [2].

We say that a function $h\colon \omega \to \omega \setminus \{0\}$ is an *order* (Schnorr) if h is computable, nondecreasing and $\lim_s h(s) = \infty$. We say that a function $f : \omega \to \omega$ is *computably traceable* with respect to the order h if there is a computable sequence $\langle F_x \rangle_{x<\omega}$ of finite sets such that for all x, $|F_x| \le h(x)$ and $f(x) \in F_x$. We will say that a degree **a** is computably traceable iff there is some order h such that every f of degree **a** or less can be computably traced with respect to h. Finally, we will say that **a** is *strongly* computably traceable iff it is computably traceable with respect to any order. Here the idea is that the real is *computationally feeble*, in the sense that we have very good approximations to computations using A as an oracle. Perhaps one would expect that such reals would be highly non-random.

We have shown:

Theorem 0.1 (Cholak et al. [1])**.** *Every c.e. strongly jump-traceable set is K-trivial.*

Thus for the first time, we have an example of a combinatorial property that at least *implies* K-triviality. The proof of this result relies on a new combinatorial technique using a kind of amplification of the traceability along the lines of the decanter or golden run method. It is beyond known technology; we believe that it could have other applications within computability theory and randomness.

On the other hand we also prove the following.

Theorem 0.2 (Cholak et al. [1])**.** *There is a K-trivial c.e. set that is* not *strongly jump-traceable. Indeed it is not jump traceable with a bound of size roughly* $\log\log n$.

This is the first example of a class defined by cost functions which we know does not coincide with the K-trivials in the proof technique is novel, since it is the first time a cost function has been used which still allows for the defeat of one involving Kolmogorov complexity.

References

[1] Peter Cholak, Rod Downey, and Noam Greenberg. Strong jump-traceability and K-triviality. 0.1, 0.2

[2] André Nies. Lowness properties and randomness. *Adv. Math.*, 197(1):274–305, 2005. ISSN 0001-8708. (document)

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