

# EQUICONSISTENCY OF CHOICELESS HIGHER CHANG CONJECTURES WITH ONE ERDŐS CARDINAL

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We are interested in the connection between cardinals defined by certain combinatorial properties and cardinals satisfying certain model theoretic relations. In particular we will look at Erdős cardinals and generalised Chang conjectures.

Even though with the axiom of choice (AC) several versions of the Chang conjecture have consistency strengths in the realm of strong cardinals, higher or are even inconsistent, without AC we show that some versions of the Chang conjecture are equiconsistent with just one Erdős cardinal.

By a Chang conjecture we mean that for some cardinals  $\kappa, \kappa', \lambda$  and  $\lambda'$ , the statement  $\langle \kappa, \lambda \rangle \rightarrow \langle \kappa', \lambda' \rangle$  holds, i.e., for any structure with domain  $A$  of size  $\kappa$  with a unary predicate  $B \subseteq A$  of size  $\lambda$  in a countable language, there is an elementary substructure  $\langle A', B', \dots \rangle \prec \langle A, B, \dots \rangle$  with  $|A'| = \kappa'$  and  $|B'| = \lambda'$ . On the other hand, a cardinal  $\kappa$  is  $\lambda$ -Erdős if it is the least such that  $\kappa \rightarrow (\lambda)^{<\omega}$  holds, i.e., for every partition  $f$  of the finite subsets of  $\kappa$  into two colours, there is a homogeneous set for  $f$  of size  $\lambda$ .

We show that for  $\lambda$  a regular cardinal, the existence of a  $\lambda$ -Erdős cardinal  $\kappa$  is equiconsistent with  $\langle \lambda^+, \lambda \rangle \rightarrow \langle \lambda, \nu \rangle$  for any infinite  $\nu < \lambda$ . For the forward direction we construct a symmetric model where  $\kappa$  is  $\lambda^+$  and still  $\lambda$ -Erdős. In that model,  $\langle \lambda^+, \lambda \rangle \rightarrow \langle \lambda, \nu \rangle$  holds for all  $\nu$  with  $\lambda > \nu \geq \omega$ . For the converse we use Jensen's indiscernibles lemma to show that if one of these versions of the Chang conjecture holds, then  $(\lambda^+)^V$  is  $\lambda$ -Erdős in  $K$ , the Dodd-Jensen core model.

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