Complexity of fuzzy predicate logics with witnessed semantics – abstract

Petr Hájek

Investigated are continuous t-norm based fuzzy predicate logics with standard semantics (the set of truth degrees is the closed real unit interval), as introduced in my Metamathematics of fuzzy logic (Kluwer 1998); in particular, Łukasiewicz, Gödel and product logic. Semantics is Tarski-style; over a crisp non-empty domain predicates are interpreted by fuzzy ([0,1]-valued) relations, the value of a universally quantified formula being the infimum of the values of instances and analogously for existential quantification and supremum. A tautology is a formula having the value 1 in each interpretation; a formula is satisfiable if it has the value 1 in some interpretation. The arithmetical complexity of the set of tautologies and the set of satisfiable formulas is known to be: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete; for product logic non-arithmetical, non arithmetical. An interpretation is witnessed if the truth value of each universally quantified formula is the minimum of truth values of instances (the infimum is taken) and the truth value of each existentially quantified formula is the maximum of values of instances. A witnessed tautology is a formula having the value 1 in each witnessed interpretation; similarly for witnessed satisfiability. Now the complexity of the set of witnessed tautologies / witnessed satisfiables: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete (for both logics the same complexity as without the assumption of witnessedness); for product logic the set of witnessed tautologies is Π_2-hard (nothing more is known) whereas the set of witnessedly satisfiable formulas is Π_1-complete. For Gödel and product the set of witnessedly satisfiable formulas is equal to the set of formulas in the classical Boolean logic. Much more is known; a survey will be given.

References

[1] P. Hájek: Metamathematics of fuzzy logic, Kluwer, 1998.

[2] P. Hájek: On witnessed models in fuzzy logic. Mathematical Logic Quaterly 53 (2007) 66-77

[3] P. Hájek: On witnessed models in fuzzy logic II. Submitted.