BACK AND FORTH BETWEEN KRIPKE MODELS

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The question whether given two structures validate the same sentences arises in a natural way in investigations of semantics of logical systems. Usually, the general answer is not obvious, if only we discard the notion of the structure isomorphism as a too restrictive one. We consider the above question in the context of Kripke semantics for intuitionistic first-order theories and introduce the notion of bounded bisimulation for first-order Kripke models.

In short, a Kripke model $\mathcal{K}$ for a given first-order language L can be viewed as a functor from a partial order $\mathbb{A}$ (viewed as a small category) to the category $\mathbb{M}$ of classical first-order structures over L whose arrows are weakly structure preserving morphism. The elements of $\mathbb{A}$ are called the nodes, and the classical structure assigned to a node α is denoted by $\mathcal{K}_\alpha$.

The bounded bisimulation for first-order Kripke models is defined as a ternary relation that may hold between two nodes α, β of Kripke models $\mathcal{K}$ and $\mathcal{M}$ respectively, and a (finite) map π from the domain of $\mathcal{K}_\alpha$ to the domain of $\mathcal{M}_\beta$. The definition involves certain back-and-forth conditions concerning both the nodes of the Kripke models, and the domains of their underlying classical structures. In particular, π will satisfy certain (finite) extension properties with respect to $\mathcal{K}_\alpha$ and $\mathcal{M}_\beta$, and structures that are related to $\mathcal{K}_\alpha$ and $\mathcal{M}_\beta$ by appropriate morphisms. We prove that if α and β are bisimilar via π to some degree c, then the domain of π satisfies at the node α exactly the same formulas of a given complexity related to c, as the image of π does at the node β. The above mentioned theorem can be viewed as a first-order Kripke model variant of the Ehrenfeucht-Fraïssé Theorem for the notion of n-partial isomorphism and the class of formulas of quantifier complexity n, viewed as the maximal number of nested quantifiers. In our theorem the complexity of a first-order formula takes into account the number of nested implications, and numbers of nested universal and existential quantifiers.

We prove basic properties of the bounded bisimulation defined above. In particular, we show that the bounded bisimulation can be described in terms of n-extendible partial isomorphism (e.g., in terms of finite Ehrenfeucht-Fraïssé games) between the appropriate classical structures of the models in question. This implies that our notion of bounded bisimulation comprises, as a particular case, the first-order bisimulation in the sense of [1]. Our results suggest how the standard techniques of classical model theory can be applied in investigation of properties of Kripke models. To illustrate this phenomena, we consider the notion of Kripke elementary submodel, and turn to the problem of constructing bounded elementary Kripke submodels of a given Kripke model. We show how this problem can be solved in the class of Kripke models over the category of ω-saturated classical structures.

References

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