Automatic linear Orders

A structure $\mathfrak{A} = (A, R_1, \ldots, R_n)$ is automatic if its domain A and all its relations R_i are finite automaton recognisable (automatons for relations working synchronously on tuples of finite words). The structure has an automatic presentation if it is isomorphic to some automatic structure.

It is known that there exists an algorithm that given a relation which is first order definable (with parameters) in automatic structure with an additional quantifier $\exists^{\infty}$ constructs an automaton recognising this relation. Hence the first order theory with this additional quantifier of an automatic structure is decidable [1, 5].

Our investigations concern the recursive isomorphism problem for two automatic presentations of linear order [3, 4].

Delhomme achived the next result:

Theorem 1. **[2]** *An ordinal α is automatically presentable if and only if $\alpha < \omega^{\omega}$.*

This fact was generalized by S. Rubin and B. Khoussainov [5]. If we factor a linear order by the equivalence relation «there is a finite number of elements between x and y» then FC-rank of the linear order is a number of such factorisations after which we get a dense order or **1**-order (order with 1 element) from the given order.

Theorem 2. **[5]** *The FC-rank of every automatic linear order is finite.*

Thus the next results were achived:

Theorem 3. *Every two automatic presentations of ordinal $\alpha < \omega^{\omega}$ are recursive isomorphic.*

The linear order is scattered if it does not contain a nontrivial dense subordering

Theorem 4. *Every two automatic presentations of automatic scattered linear order with FC-rank 2 are recursive isomorphic.*

Theorem 5. *Every automatic linear order is definable in appropriate automatic linear order with FC-rank 1.*

In addition examples of non-periodic automatic linear order were provided. The linear order is periodic if it is $\sum\limits_{i\in\omega^*} \mathcal{A}_i + \mathcal{B} + \sum\limits_{i\in\omega} \mathcal{C}_i$, where $\mathcal{A}_i = \mathcal{A}_j = \mathcal{A}$ for all $i \in \omega^*$, $\mathcal{C}_i = \mathcal{C}_j = \mathcal{C}$ for all $i \in \omega$ and $\mathcal{A}, \mathcal{B}, \mathcal{C}$ — some linear orders.

References

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