Provably Recursive Functions in Extensions of a Predicative Arithmetic

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We provide a short summary of recent research into extensions of the weak first order theory of arithmetic $EA(I;O)$ developed by S.Wainer and previous research students, e.g. [2]. $EA(I;O)$ replaces the full induction scheme of PA by what can be seen as a predicative induction rule, incorporating the idea of variable separation from Bellantoni & Cook [1]. The principle effect is that the provably recursive functions are now the Kalmar elementary functions with the bounding functions in the ordinal analysis coming from the slow growing hierarchy.

In this talk we will present a conservative extension of $EA(I;O)$ which provides a more natural approach to composition of provably recursive functions. Furthermore, this theory can be easily extended into a hierarchy to capture higher levels of the Grzegorczyk hierarchy. The difficultly in proving so comes about in establishing upper bounds. We look in detail at the first level above our base theory where we can employ a similar finitary analysis as used on $EA(I;O)$ in [3]. In doing so we raise a number of interesting issues such as composition of the slow growing G_α functions.

[1] S. Bellantoni and S. Cook, "A new recursion theoretic characterization of the polytime functions", Computational Complexity Vol. 2 (1992) pp. 97-110.

[2] N. Cagman, G. E. Ostrin and S. S. Wainer, "Proof theoretic complexity of low subrecursive classes", in F. L. Bauer and R. Steinbrueggen (eds) Foundations of Secure Computation, IOS Press Amsterdam 2000, pp. 249-286.

[3] G. E. Ostrin and S. S. Wainer, "Elementary arithmetic", Annals of Pure and Applied Logic 133 (2005) pp. 275-292.