Title: Fragments of Martin's axiom related to the rectangle refining property

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A partition $[\omega_1]^2 = K_0 \cup K_1$ has the rectangle refining property if for any $I, J \in [\omega_1]^{\aleph_1}$, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $\alpha \in I'$ and $\beta \in J'$ with $\alpha < \beta$, $\{\alpha, \beta\} \in K_0$. This property has been defined by Larson-Todorčević to solve Katětov's problem.

In 1980's, Stevo Todorčević has studied fragments of Martin's axiom. Let $\mathsf{MA}_{\aleph_1}$ be Martin's axiom for $\aleph_1$-many dense sets of ccc forcing notions, $\mathcal{K}_2$ the statement that every ccc forcing notion has the property K, $\mathcal{C}^2$ the statement that any product of ccc forcing notions still has the ccc. We note that $\mathsf{MA}_{\aleph_1}$ implies $\mathcal{K}_2$, and $\mathcal{K}_2$ implies $\mathcal{C}^2$. However it has been unknown whether these reverse implications hold.

In this talk, we consider new chain condition of forcing notions. A forcing notion $\mathbb{P}$ has the anti-rectangle refining property if it is uncountable and for any $I, J \in [\mathbb{P}]^{\aleph_1}$, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $p \in I'$ and $q \in J'$, p and q are incompatible in $\mathbb{P}$. Let $a(\mathbb{P})$ be the forcing notion adding an antichain in $\mathbb{P}$ by finite approximations. If a forcing notion $\mathbb{P}$ has the anti-rectangle refining property, then for any $I, J \in [a(\mathbb{P})]^{\aleph_1}$ with $I \cup J$ pairwise disjoint, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $\sigma \in I'$ and $\tau \in J'$, σ and τ are compatible in $a(\mathbb{P})$, that is, $\sigma \cup \tau \in a(\mathbb{P})$. This property is stronger than the countable chain condition. Let $\mathsf{MA}_{\aleph_1}(a(\text{arec}))$ be the $\mathsf{MA}_{\aleph_1}$ for all forcing notions $a(\mathbb{P})$ such that $\mathbb{P}$ has the anti-rectangle refining property.

We can show that it is consistent that $\mathsf{MA}_{\aleph_1}(a(\text{arec}))$ holds but $\mathcal{C}^2$ fails, etc.