(1) Determine all points at which relative maxima and minima as well as points of inflection occur, as well as all intervals on which the function is increasing or decreasing, concave up or concave down, for the function

\[ f(x) = x^3 + 5x - 12 \]

(2) Find the tangent line at \( x = 1 \) to the graph of the function

\[ g(x) = (e^{-2x} + 1)(\ln x - \frac{1}{\sqrt{x}}) \]

(3) Find

\[ \frac{d}{dx} \left[ \frac{\sqrt{x^2 - 1} + 1}{(1 - 2x)^3} \right] \]

(4) The amount of bacteria in a lab dish (in grams) after \( t \) hours is

\[ B(t) = 10 \cdot 2^{\frac{1}{3}t} \]

Use approximation by increments to estimate the amount of bacteria added during the the fourth hour. (Use \( \ln 2 \approx .7 \))

(5) The population of a city (in thousands, after \( t \) years) is given by the function

\[ P(t) = 4 + \frac{20e^t}{e^t + 1} \]

Find the population of the city “in the long run”.

(6) The price \( p \) of a new phone and the cost of advertising \( c \) per phone sold (both in dollars) are related by the equation \( p^2 + 5pc + 10c^2 = 1240 \). If the price increases by $2 each year, at what rate is the cost of advertising changing when the price is $30? Is the cost of advertising increasing or decreasing at that moment?

(7) From the definition of the derivative, compute

\[ \frac{d}{dx} \sqrt{6 - x} \]

at \( x = 2 \).
(8) Determine for the function

\[ f(x) = \frac{x^2 - 4}{x - 3} \]

the domain, all asymptotes, the \( x \)-values of all relative maxima, relative minima and points of inflection, as well as all intervals on which \( f \) is increasing, decreasing, concave up and concave down.

(9) The population of a town after \( t \) years, and the waste (in kg) produced by each person, are given by

\[ P(t) = 1000e^{\frac{1}{4}t^4} \quad \text{and} \quad W(t) = 100e^{-t^2} \]

Find the times over the first two years at which the total waste produced by the town is minimal, and at which it is maximal.

(9) Find \( f' \) by logarithmic differentiation for

\[ f(x) = \frac{(x + 4)^5\sqrt{x^2 - 1}}{x^3e^x} \]

(Note: You do not need to simplify your answer.)

(10) Compute the integrals

\[ \int \frac{x^3}{\sqrt{x^2 + 1}} \, dx \quad \text{and} \quad \int x^2e^{-x} \, dx \]

(11) Suppose the wind speed at noon is 50km/h and decreasing at the rate of \( 10 - 3t \) km/h per hour (where \( t \) is the number of hours after noon) over the next three hours. What is the wind speed at 3 p.m.? What is the average temperature during those three hours?

(12) A hit-and-run accident was witnessed by 5% of a town, and three hours later, 15% of the town had heard about it. Suppose the number of town residents who had heard about it after \( t \) hours is given by the equation

\[ N(t) = \frac{P}{1 + Ce^{-kt}} \]

where \( P \) is the population of the town and \( C \) and \( k \) are constants. Find \( C \) and \( k \), the time at which half the town’s residents had heard about it, and the time at which the news about the accident is spreading the most rapidly.

(13) Find the particular solution to the differential equation

\[ \frac{dy}{dx} = y \ln x \]

when \( y(1) = 0 \).