Math 234 **Name: _____**
Sample Exam 1
Time: 75 minutes

Instructions: Show all work.
A non-programmable calculator
and a two sided 8×11 formula sheets are allowed.

Problem	Maximum points	Score
1	25	
2	10	
3	12	
4	25	
5	8	
Total	80	

Problem 1: A particle is moving with its position vector at time *t* given by:

$$\mathbf{r} = (\sin t \cos t)\mathbf{i} + 5\mathbf{j} + (\tfrac{1}{2})(\cos^2 t - \sin^2 t)\mathbf{k}$$

a) (6 pts) Show that its speed $|\mathbf{v}| = 1$.

b) (4 pts) Find the unit tangent vector **T** at any time t.

Answer:

c) (5 pts) Find the curvature κ at any time t.

Answer:

d) (5 pts) Find the unit normal vector **N** at any time t.

Answer:

e) (5 pts) Show that the given curve:

$$\mathrm{r} = (\sin t \cos t)\mathbf{i} + 5\mathbf{j} + (\tfrac{1}{2})(\cos^2 t - \sin^2 t)\mathbf{k},$$

is a circle centered at (0, 5, 0) with radius ½.

Problem 2: Let $f(x,y) = 8 - x^2 - y^2$

a) (5 pts) Describe the level curves of *f*, and sketch two of them.

b) (5 pts) Sketch the graph of *f*.

Problem 3: a) (4 pts) Find all second order partial derivatives of the function:

$$f(x,y) = x^2 \sin(2x - 3y)$$

b) Let $g(x,y)$ be a function defined everywhere and satisfies: $g(x,0) = 1$. Which of the following must be true? Give reasons for your answer.

i) **(4 pts):** $\dfrac{\partial g}{\partial x}(0,0) = 0.$ **True** **False** (Circle one)

Reason:

ii) **(4 pts):** $\dfrac{\partial g}{\partial y}(0,0) = 0.$ **True** **False** (Circle one)

Reason:

Problem 4: Consider the function:

$$f(x,y)=\frac{(x-y)^2}{\sqrt{x^2+y^2}}$$

a) (3 pts) Find the domain *Dom*(*f*).

Answer:

b) (3 pts) Is *Dom*(*f*) open? Why, why not?

Answer:

c) (3 pts) Is *Dom*(*f*) closed? Why, why not?

Answer:

d) (3 pts) Is *Dom*(*f*) bounded? Why, why not?

Answer:

For the same function: $f(x,y)=\dfrac{(x-y)^2}{\sqrt{x^2+y^2}}$

e) (5 pts) Show that $\lim_{(x,y)\to(0,0)} f(x,y)=0$

Hint: Use polar coordinates.

f) (3 pts) Where is $f(x,y)$ continuous? Justify your answer.

Answer:

g) (5 pts) Find the range of $f(x,y)$. Justify your answer.

Answer:

Problem 5: (8 pts) Let **v** and **a** be the velocity and acceleration vectors of a moving particle. Show that the unit tangent and normal vectors (if they exist) must satisfy:

$$\mathbf{T}\times\mathbf{N}=\frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

Hint: Start with $\mathbf{v}=|\mathbf{v}|\mathbf{T}$.

Best of luck

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