$$x' = x - 5y + \cos 2t$$
$$y' = x - y$$

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} \cos 2t \\ 0 \end{bmatrix}$$

$$(1-\lambda)(-1-\lambda) + 5 = 0$$
$$-1 + \cancel{\lambda} - \cancel{\lambda} + \lambda^2 + 5 = 0$$

$$\lambda^2 + 4 = 0 \qquad \lambda = \pm 2i$$

$$\begin{bmatrix} 1-2i & -5 \\ 1 & -1-2i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(1-2i)\xi_1 - 5\xi_2 = 0$$

$$\xi_1 = 1 + 2i$$

$$(1-2i)(1+2i) - 5\xi_2 = 0$$

$$(1+4) - 5\xi_2 = 0 \qquad \xi_2 = 1$$

$$\begin{bmatrix} 1+2i \\ 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 5 \\ (1-2i) \end{bmatrix}$$

$$\underline{x}_h = C_1 \begin{bmatrix} 5 \\ (1-2i) \end{bmatrix} e^{2it} + C_2 \begin{bmatrix} 5 \\ 1+2i \end{bmatrix} e^{-2it}$$

$$\underline{x}^{(1)} = \begin{bmatrix} 5 \\ 1-2i \end{bmatrix} \{ \cos 2t + i \sin 2t \}$$

$$Re\,(\underline{x}^{(1)}) = \begin{bmatrix} 5\cos 2t \\ 2\sin 2t \end{bmatrix} + \begin{bmatrix} \\ \cos 2t \end{bmatrix}$$

$$Im\,(\underline{x}^{(1)}) = \begin{bmatrix} 5\sin 2t \\ -2\cos 2t + \sin 2t \end{bmatrix}$$

$$\underline{x}_h = C_3 \begin{bmatrix} 5\cos 2t \\ 2\sin 2t + \cos 2t \end{bmatrix} + C_4 \begin{bmatrix} 5\sin 2t \\ -2\cos 2t + \sin 2t \end{bmatrix}$$

$$\underline{x}_p = \underbrace{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}}_{\underline{a}} t\cos 2t + \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\underline{b}} t\sin 2t + \underbrace{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}_{\underline{c}} \cos 2t + \underbrace{\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}}_{\underline{d}} \sin 2t$$

$$\underline{a}\cos 2t \oplus -2\underline{a}t\sin 2t + \underline{b}\sin 2t + 2\underline{b}t\cos 2t$$

$$\oplus -2\underline{c}\sin 2t + 2\underline{d}\cos 2t$$

$$= \underline{A}\left\{\underline{a}t\cos 2t + \underline{b}t\sin 2t + \underline{c}\cos 2t + \underline{d}\sin 2t\right\} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\cos 2t$$

$t\cos 2t$: $\quad 2\underline{b} = \underline{A}\,\underline{a}$ $\quad$ ①

$t\sin 2t$: $\quad -2\underline{a} = \underline{A}\,\underline{b}$ $\quad$ ②

$\cos 2t$: $\quad \underline{a} + 2\underline{d} = \underline{A}\,\underline{c} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad$ ③

$\sin 2t$: $\quad \underline{b} - 2\underline{c} = \underline{A}\,\underline{d}$ $\quad$ ④