Math 320 Spring 2016 Preliminaries

We will study Ordinary Differential Equations (ODEs) using the tools of Linear Algebra (LA).

Major Topics

1. First-order ODEs (some review!)
   * significance of linearity
   * why systems of 1^{st}-order ODEs
   * connection to higher-order ODEs
   * connection to Partial Differential Equations (PDEs)

$$\left.\begin{array}{l}\text{2. Tools of Linear Algebra}\\ \text{3. Higher-order linear ODEs}\\ \text{4. Systems of } 1^{st}\text{-order ODEs}\end{array}\right\}\ \text{using LA}$$

Some Definitions (Review !)

A **Differential Equation** is an equation containing one or more derivatives of the dependent variable.

$y(x)$; y is the dependent variable
x is the independent variable

$x(y,t)$; x is the dependent variable
y, t are independent variables

DEs arise in science and engineering when we study rate of change of some quantity (e.g. Temperature) as a function of space or time or both.

Ordinary DEs The dependent variable is a function of only one independent variable and all derivatives are ordinary

e.g. growth in time of a species with population $P(t)$:

$$\frac{dP(t)}{dt} = A\,P(t) \quad , \quad A > 0 \text{ constant}$$

e.g. Newton's Law $\underline{F}(t) = m\,\underline{a}(t) = m\dfrac{d^2\underline{x}(t)}{dt^2}$

where $\underline{x}(t)$ is the vector position of a particle as a function of time t, and $\underline{F}(t)$ is a prescribed force.

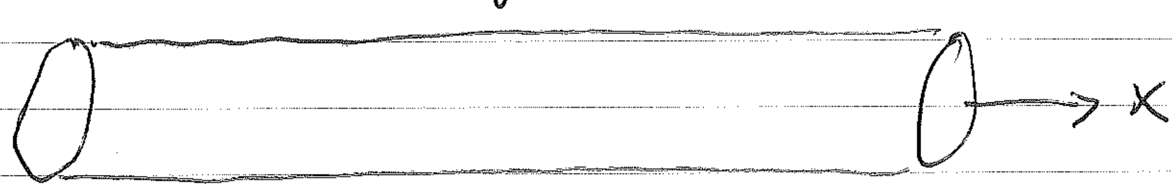
$\boxed{\text{Partial DEs}}$ The dependent variable is a function of more than one independent variable and derivatives are partial derivatives.

e.g. the Heat Equation (Diffusion Equation)

$$\frac{\partial T(x,t)}{\partial t} = \nu \frac{\partial^2 T(x,t)}{\partial x^2}$$

$T(x,t)$ is the temperature at a point x and time t (one space dimension)

This equation describes how heat spreads (diffuses) in a homogeneous material



The $\boxed{\text{Order}}$ of a DE is the order of the highest derivative

Population growth $\frac{dP(t)}{dt} = A\,P(t)$ is a 1^{st}-order ODE

Newton's Law $m\frac{d^2\underline{x}(t)}{dt^2} = \underline{F}(t)$ is a 2^{nd}-order ODE

The Heat Equation $\frac{\partial T(x,t)}{\partial t} = \nu\frac{\partial^2 T(x,t)}{\partial x^2}$ is a 2^{nd}-order PDE.

$\boxed{\text{Linearity}}$ The dependent variable and all of its derivatives appear to the power 1.

All these examples are linear!

A linear ODE may be written as

$$a_0(x)\, y^{(n)}(x) + a_1(x)\, y^{(n-1)}(x) + \cdots + a_n(x)\, y(x) = F(x)$$

where superscript (n) means derivative:

$$y^{(n)}(x) = \frac{d^{(n)} y(x)}{dx^{(n)}} \quad , \quad y^{(n-1)}(x) = \frac{d^{(n-1)} y(x)}{dx^{(n-1)}}$$

The coefficients $a_0(x), a_1(x), \ldots, a_n(x), F(x)$

can be nonlinear because linearity of the ODE is with respect to the dependent variable (and its derivatives).

A Nonlinear ODE is anything else.

Examples

$$x \frac{dy(x)}{dx} - \sin^3(x) = -x^2 \frac{d^2 y(x)}{dx^2} - 2y(x)$$

is a linear, 2^{nd}-order ODE

$\frac{dy(x)}{dx} + \sin(y) = 0$ 1st-order, nonlinear ODE

$y'(x) = \frac{x}{y(x)}$ 1st-order nonlinear ODE

$e^x \frac{d^3 y(x)}{dx^3} = e^{x^2} y(x) + x^3 \sin^3(x)$

3rd-order linear ODE

Come to class with examples of

* a 2nd-order nonlinear ODE
* a 1st-order nonlinear PDE
* a 4th-order linear ODE

* Do your equations have physical significance?