MATH320 Problem set 4,5 Solutions
written by Jean

Section 3.4 19.

$$\begin{pmatrix} 1 & 0 & 0 & 3 & -1 & 0 \\ 0 & 1 & 0 & -2 & 6 & 0 \\ 0 & 0 & 1 & 1 & -8 & 0 \end{pmatrix}$$

This is already in the reduced echelon form. x_4, x_5 are free variables. So, we set $x_4 = s$ and $x_5 = t$. Then,

$$\begin{aligned} x_1 &= -3s + t \\ x_2 &= 2s - 6t \\ x_3 &= -s + 8t \\ x_4 &= s \\ x_5 &= t \end{aligned}$$

Answer:

$$\mathbf{x} = \begin{pmatrix} -3s+t \\ 2s-6t \\ -s+8t \\ s \\ t \end{pmatrix} = s\begin{pmatrix} -3 \\ 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -6 \\ 8 \\ 0 \\ 1 \end{pmatrix} \quad \text{with} \quad s, t \in \mathbb{R}$$

Section3.4 32.(a) Since,

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= A^2 + AB + BA + B^2, \end{aligned}$$

it suffices to show that $AB \neq BA$ to verify that $(A+B)^2 \neq A^2 + 2AB + B^2$. One can explicitly check that

$$AB = \begin{pmatrix} -1 & 3 \\ 5 & 1 \end{pmatrix} \qquad BA = \begin{pmatrix} -18 & 14 \\ -22 & 18 \end{pmatrix}$$

So, $AB \neq BA$.$\square$
(b)If $AB = BA$, then

$$\begin{aligned} (A+B)^2 &= A^2 + AB + BA + B^2 \\ &= A^2 + AB + AB + B^2 \\ &= A^2 + 2AB + B^2 \end{aligned}$$

Section 3.6 30.(Cramer's rule)

$$x_1 = \frac{\begin{vmatrix} 3 & 4 & 2 \\ 1 & 2 & 1 \\ 3 & -2 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & 2 \\ 4 & 2 & 1 \\ 2 & -2 & -5 \end{vmatrix}} = -\frac{8}{56} = -\frac{1}{7}$$

$$x_2 = \frac{\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 3 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & 2 \\ 4 & 2 & 1 \\ 2 & -2 & -5 \end{vmatrix}} = \frac{78}{56} = \frac{39}{28}$$

$$x_3 = \frac{\begin{vmatrix} 1 & 4 & 3 \\ 4 & 2 & 1 \\ 2 & -2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & 2 \\ 4 & 2 & 1 \\ 2 & -2 & -5 \end{vmatrix}} = -\frac{68}{56} = -\frac{17}{14}$$

If you find a typo or miscalculations, please let us know by sending emails.
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