Problem set 6

4.2 #7 The set W of all vectors in $\mathbb{R}^2$ such that $|x_1| = |x_2|$ is NOT a subspace of $\mathbb{R}^2$. It is not closed under addition. For example $(1,-1)$ and $(-2,-2)$ are two vectors in W but their sum $(-1,-3)$ is not in W (b/c $|-1| \neq |-3|$)

4.2 #8 The set W of all vectors in $\mathbb{R}^2$ such that $(x_1)^2 + (x_2)^2 = 0$ is a subspace of $\mathbb{R}^2$. In fact W is the singleton set that consists of the zero-vector $W = \{(0,0)\}$.

4.2 #17

The augmented coefficient matrix is

$$\left[\begin{array}{cccc|c} 1 & 3 & 8 & -1 & 0 \\ 1 & -3 & -10 & 5 & 0 \\ 1 & 4 & 11 & -2 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-R_1} \left[\begin{array}{cccc|c} 1 & 3 & 8 & -1 & 0 \\ 0 & -6 & -18 & 6 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right] \xrightarrow[R_2+6R_3]{R_1-3R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The leading variables are x_1 & x_2
x_3 & x_4 are free variables

$$x_4 = t, \quad x_3 = s, \quad x_2 = -3s + t, \quad x_1 = s - 2t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

4.2 #27 * W contains the zero vector $0 = 0 \times \mathbf{U} + 0 \times \mathbf{V}$

* W is closed under addition

because given $a_1\mathbf{U} + b_1\mathbf{V}$ and $a_2\mathbf{U} + b_2\mathbf{V}$ in W, their sum $(a_1+a_2)\mathbf{U} + (b_1+b_2)\mathbf{V}$ is also in W

* W is closed under scalar multiplication

Given $a\mathbf{U} + b\mathbf{V}$ in W and a scalar c, $c(a\mathbf{U} + b\mathbf{V}) = ca\mathbf{U} + cb\mathbf{V}$ is also in W

So, W is a subspace.

Problem set 6

3.6 #52

$A^T = A^{-1}$ implies that $\det(A^T) = \det(A^{-1})$

using the fact that $\det(A^T) = \det(A)$ and $\det(A^{-1}) = \frac{1}{\det(A)}$

we get $\det(A) = \frac{1}{\det(A)} \iff (\det(A))^2 = 1 \implies \det(A) = \pm 1$

3.7 #8

We need to find the 3-degree polynomial that ~~goes~~ fits the points $(-1,3), (0,5), (1,7)$ and $(2,3)$

Let $y = a + bx + cx^2 + dx^3$

When we plug in the points we get the following linear system

$$\begin{aligned} 3 &= a - b + c - d \\ 5 &= a \\ 7 &= a + b + c + d \\ 3 &= a + 2b + 4c + 8d \end{aligned} \iff \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \\ 3 \end{bmatrix}$$

The augmented coefficient matrix is

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 3 \\ 1 & 0 & 0 & 0 & 5 \\ 1 & 1 & 1 & 1 & 7 \\ 1 & 2 & 4 & 8 & 3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 1 & -1 & 1 & -1 & 3 \\ 1 & 1 & 1 & 1 & 7 \\ 1 & 2 & 4 & 8 & 3 \end{array}\right]$$

$$\xrightarrow[\substack{R_3 - R_1 \\ R_4 - R_2}]{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & -1 & 1 & -1 & -2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 3 & 3 & 9 & 0 \end{array}\right] \xrightarrow[R_4 - 3R_3]{R_3 + R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & -1 & 1 & -1 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & -6 \end{array}\right]$$

$\implies d = -1, \; c = 0, \; b = 3$ and $a = 5$

so, $y = 5 + 3x - x^3$