Practice Problems for Final Exam

1. Find a formula for the general solution in terms of the fundamental matrix and its inverse. You should find explicitly the fundamental matrix and its inverse.

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} -4 & -2 \\ 0.5 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \exp(-3t) \\ t \end{bmatrix} \tag{1}$$

2. Solve, and state the region of validity of the solution.

$$(x^2 - 4)y' + 2y = (x^2 - 4)(2 - x)^{-1/2}, \quad y(1) = 1 \tag{2}$$

3. Use Variation of Paramters to find the general solution to

$$\frac{x^2}{2}y'' + 2xy' - \frac{7}{8}y = \frac{x^4}{2}, \quad x > 0. \tag{3}$$

4. Note: You can do most of parts (b) and (c) without part (a).

(a) Find a basis for the subspace spanned by the columns of $\mathbf{A}$, where $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 5 & -2 & 2 \\ 3 & -2 & 1 \\ -1 & 6 & 1 \end{bmatrix}. \tag{4}$$

(b) Find b_3 such that $\mathbf{Ax} = \mathbf{b}$ has solutions, with $\mathbf{A}$ given above and $\mathbf{b}^T = [3, 4, b_3]$. Show that $\mathbf{b}$ with this value of b_3 is a linear combination of the basis vectors found in (a).

(c) Solve $\mathbf{Ax} = \mathbf{b}$ for $\mathbf{b}^T = [3, 4, b_3]$ and the value of b_3 found in (b).

5. Solve, and express the solution in terms of real functions.

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 3 & -5 \\ 1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}^T(\pi/4) = [2 \;\; 4] \tag{5}$$

6.

(a) Write down the form of the particular solution for the system:

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} -4 & -5 \\ 5 & 4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \exp(-3t) \\ \cos(3t) \end{bmatrix} \tag{6}$$

(b) Write down the equations for the vector coefficients of your particular solution. You do not need to solve these equations.