

Math 320 (Smith): Practice Exam 2

1. (20 min.)

(a) (20 points) For what vectors \mathbf{b} does $\mathbf{Ax} = \mathbf{b}$ have a solution, with \mathbf{A} given by

$$\mathbf{A} = \begin{bmatrix} 6 & 3 & 3 \\ 2 & 5 & -1 \\ -4 & -8 & 1 \end{bmatrix} \quad (1)$$

(b) Find all possible solutions for $\mathbf{b}^T = [0 \ 1 \ -3/2]$.

2. (15 min.) Solve

$$4y'' + 4y' + y = 0 \quad (2)$$

for $y(4) = \exp(-2)$, $y'(4) = 2\exp(-2)$. Show your work (no work, no credit).

3. (15 min) Use the method of Reduction of Order to find the general solution to

$$4x^2y'' + y = 0, \quad x > 0 \quad (3)$$

given one solution $y_1(x) = x^{1/2}$. Show your work (no work, no credit).

4. (15 min) Find the general solution to

$$\frac{d^7y}{dx^7} + 2\frac{d^5y}{dx^5} + \frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + y = 0 \quad (4)$$

given that the characteristic equation factors into $(r^3 + 1)(r^2 + 1)^2 = 0$. If possible, express your solution in terms of real functions. Show your work (no work, no credit).

Note: You should be able to do this problem after lecture on April 1.

5. (5 min) Are the functions $y_1(x) = 5x$, $y_2(x) = 5(x + 20)$ and $y_3(x) = x + \exp(x)$ linearly dependent or linearly independent? Justify your answer (no work, no credit).