Math 320 (Smith): Practice Exam 2

1. (a) For what vectors $\mathbf{b}$ does $\mathbf{Ax} = \mathbf{b}$ have a solution, with $\mathbf{A}$ given by

$$\mathbf{A} = \begin{bmatrix} 6 & 3 & 3 \\ 2 & 5 & -1 \\ -4 & -8 & 1 \end{bmatrix} \tag{1}$$

(b) Find a basis for the vector space spanned by the columns of $\mathbf{A}$.
(c) Find all possible solutions for $\mathbf{b}^T = [0 \;\; 1 \;\; -3/2]$.

2. Solve

$$4y'' + 4y' + y = 0 \tag{2}$$

for $y(4) = \exp(-2)$, $y'(4) = 2\exp(-2)$. Show your work (no work, no credit).

3. Use the method of Reduction of Order to find the general solution to

$$4x^2y'' + y = 0, \quad x > 0 \tag{3}$$

given one solution $y_1(x) = x^{1/2}$. Show your work (no work, no credit).

4. Are the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ linearly dependent or linearly independent?

$$\mathbf{a}^T = [3 \;\; 9 \;\; 0 \;\; 5], \quad \mathbf{b}^T = [3 \;\; 0 \;\; 9 \;\; -7], \quad \mathbf{c}^T = [4 \;\; 7 \;\; 5 \;\; 0] \tag{4}$$

5. Given the equation

$$y'' + \ln(-x)\, y = 0, \quad x < 0 \tag{5}$$

Is the following statement TRUE or FALSE? Briefly explain your answer.

The Wronskian of 2 linearly independent solutions to (5) must be zero for $x < 0$.

6. Show that $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$ if $\mathbf{A}$ and $\mathbf{B}$ are arbitrary 2×2 matrices.

7. Find the determinant. Hint: Use elementary row operations.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 & 5 \\ -1 & 2 & 3 & 4 \\ 1 & 3 & 1 & -2 \\ -1 & -3 & 0 & -4 \end{bmatrix} \tag{7}$$