Math 320 (Smith): Practice Problems for Exam 2, #2

1. (a) For what vectors $\mathbf{b}$ does $\mathbf{Ax} = \mathbf{b}$ have a solution, with $\mathbf{A}$ given by

$$\mathbf{A} = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -4 & 10 \\ -1 & 6 & 0 \end{bmatrix} \tag{1}$$

(b) Find all possible solutions (or no solution) for $\mathbf{b}^T = [1 \quad -7 \quad -2]$ and for $\mathbf{b}^T = [-1 \quad 5 \quad -1]$.
(c) Find a basis for the vector space spanned by the columns of $\mathbf{A}$.

2. Solve

$$0.5y'' + 3y' + 9y = 0 \tag{2}$$

for $y(0) = 2$, $y'(0) = 5$. Show your work.

3. Find the general solution to

$$2x^2y'' + 5xy' + \frac{9}{8}y = 0, \quad x > 0 \tag{3}$$

given one solution $y_1(x) = x^{-3/4}$. Show that your general solution is constructed from two linearly independent solutions to (3).

4. Given the equation

$$x\exp(x)y'' + y = 0 \tag{4}$$

Is the following statement TRUE or FALSE?

The Wronskian of 2 linearly independent solutions to (4) is guaranteed to be different from zero for $x < 0$.

5. Find the determinant (i) using elementary row operations, and (ii) using a cofactor expansion.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 3 & 2 \end{bmatrix} \tag{5}$$

6. Find α, β such that a unique solution exists in a region at least as large as $\alpha < x < \beta$:

$$(x^2 - 4)y'' + (x - 2)y' + xy = \ln(x), \quad y(1) = -1, \quad y'(1) = 0. \tag{6}$$