

**Math 320 (Smith): Problem Set 10**

**DUE Thursday April 24, 2008**

1. EP Section 5.5: 49, 50, 54, 57, 61, 62
2. Find the general solution to

$$x^2 y'' - 3xy' + 4y = x \ln(x), \quad x > 0$$

3. Given

$$ty'' - (1+t)y' + y = t^2 \exp(2t)$$

- (a) verify that  $y_1(t) = 1 + t$  is a solution to the homogeneous problem.
  - (b) Find the general solution to the nonhomogeneous problem.
4. EP Section 5.6: 17, 25

**Hints for #3**

- A. Reduction of order with  $y(t) = v(t)(1+t)$  should give

$$\frac{v''}{v'} = \frac{1+t^2}{t(1+t)}$$

- B. Rewrite

$$\frac{1+t^2}{t(1+t)} = 1 + \frac{1}{t} - \frac{2}{(1+t)}$$

using partial fractions.

- C. Then integrate to get

$$v' = C \frac{t}{(1+t)^2} \exp(t)$$

- D. Integration by parts with  $p = t \exp(t)$  and  $dq = dt/(1+t)^2$  gives

$$\begin{aligned} v &= C \left[ \frac{-t \exp(t)}{(1+t)} + \exp(t) \right] + C_2 \\ &= \left[ \frac{C \exp(t)}{(1+t)} \right] + C_2 \end{aligned}$$

- E. Then  $y = C \exp(t) + (1+t)C_2$