

Math 320 (Smith): Problem Set 11

DUE Friday Dec 4, 2009

1. EP Section 5.5: 49, 50, 53, 61, 62

2. Find the general solution to

$$x^2y'' - 3xy' + 4y = x \ln(x), \quad x > 0$$

3. Given

$$ty'' - (1+t)y' + y = t^2 \exp(2t)$$

(a) verify that $y_1(t) = 1 + t$ is a solution to the homogeneous problem.

(b) Find the general solution to the nonhomogeneous problem.

Hints for #3

A. Reduction of order with $y(t) = v(t)(1+t)$ should give

$$\frac{v''}{v'} = \frac{1+t^2}{t(1+t)}$$

B. Rewrite

$$\frac{1+t^2}{t(1+t)} = 1 + \frac{1}{t} - \frac{2}{(1+t)}$$

using partial fractions.

C. Then integrate to get

$$v' = C \frac{t}{(1+t)^2} \exp(t)$$

D. Integration by parts with $p = t \exp(t)$ and $dq = dt/(1+t)^2$ gives

$$\begin{aligned} v &= C \left[\frac{-t \exp(t)}{(1+t)} + \exp(t) \right] + C_2 \\ &= \left[\frac{C \exp(t)}{(1+t)} \right] + C_2 \end{aligned}$$

E. Then $y = C \exp(t) + (1+t)C_2$

4. Solve problem 39 in EP Section 5.5 USING VARIATION OF PARAMETERS.

5. EP Section 5.6: 17, 25

6. EP Section 6.1: 12, 21, 29

7. EP Section 7.1: 3, 8

8. EP Section 7.1: 16, 19 (if you do not have access to a graphing calculator or computer software, try to sketch the direction field by hand).

9. EP Section 7.3: 7, 11, 21, 24, 26 (it is not necessary to do the computer graphics part of these problems).