Math 705: Problem Set 5

Due Thursday October 22, 2015

1. For atmospheric flows, one must consider the earth’s rotation and density decreasing with altitude. A model for such flow in the rotating reference frame is given by

$$\frac{Du}{Dt} + 2\Omega \hat{z} \times u = -\frac{1}{\rho_o} \nabla p - g \frac{\rho'}{\rho_o} \hat{z} + \nu_o \nabla^2 u$$  \hspace{1cm} (1)

$$\frac{D\rho'}{Dt} - bw = \kappa \nabla^2 \rho', \quad \nabla \cdot u = 0$$  \hspace{1cm} (2)

where $\Omega$ is the rotation rate and $\hat{z}$ is the rotation axis as well as the direction of density stratification. The density decreases with altitude $z$ such that the total density is given by $\rho(x,t) = \rho_o - bz + \rho'(x,t)$ with $b$ a positive constant. Here the velocity vector $u = u\hat{x} + v\hat{y} + w\hat{z}$.

(a) For the inviscid equations ($\nu_o, \kappa = 0$), show that the quantity $(\omega + 2\Omega \hat{z}) \cdot \nabla \rho$ is conserved following fluid particles, where $\omega = \nabla \times u$.

2. Please write short paragraph describing what you plan to do for the final project: provide the title/author of the paper or book chapter, briefly describe your work plan (for example, you might propose to reproduce some mathematical results, or to write a code to numerically investigate some flow equations and compare to theory).

3. Acheson problems 8.1, 8.2, 8.3, 8.5