Math 705: Problem Set 8

Due Thursday November 19, 2015

1. This problem deals with the instability of a jet (Drazin & Reid, exercise 1.7). Consider the 2D instability of a jet of thickness $2L$ in a constant-density, incompressible fluid. The base flow velocity is $u = U\hat{x}$ with $U = 0$ for $|z| > L$ and $U = U_o$ for $|z| < L$. If the disturbance has velocity potential

$$\phi' \propto \exp(st + ikx)$$

show that both

$$(s + ikU)^{-1} \frac{\partial \phi'}{\partial z} \quad \text{and} \quad (s + ikU)\phi'$$

are continuous at the interfaces $z = \pm L$.

Assuming that $\phi'$ is an odd function of $z$ (called a sinuous mode), obtain the eigenvalue relation

$$(s + ikU_o)^2 \tanh(|k|L) + s^2 = 0.$$

Deduce that this jet is unstable to all waves of all lengths.

Assuming that $\phi'$ is an even function of $z$ (called a varicose mode), obtain the eigenvalue relation

$$(s + ikU_o)^2 \coth(|k|L) + s^2 = 0.$$

[Rayleigh (1894), Lamb (1932)].

2. Continue working on your projects.