

Math 703
Methods of Applied Mathematics I
TR 1:00-2:15 in Van Vleck B119

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Textbook 1: *Introduction to Applied Mathematics*, Strang, Wellesley-Cambridge Press.

Textbook 2: *Advanced Mathematical Methods for Scientists and Engineers*, Bender and Orszag, Springer.

Prerequisites: Linear Algebra and Differential Equations.

Course description: The course introduces methods to solve mathematical problems that arise in areas of application such as physics, engineering and statistics. Roughly speaking, we can divide these problems into two categories: (i) *equilibrium*, *i.e.*, statics problems (ii) and *departures from equilibrium*, *i.e.*, dynamics problems. The first part of the course will be devoted to the study of equilibrium; linear algebra provides a unifying framework for discrete equilibrium problems from several application areas. This algebraic structure is also the basis for numerical solution of both discrete and continuous equilibrium systems. In the continuous case, equilibrium mechanics leads to boundary value problems for differential equations: in one dimension, one finds ordinary differential equations, *e.g.*, Sturm-Liouville problems; for higher dimensional systems, one finds partial differential equations, *e.g.*, Laplace's equation, Poisson's equation and the equations for Stokes flow.

Dynamics problems become initial value problems for ordinary and partial differential equations. Thus we will review some well-known techniques for solving differential equations, *e.g.*, Green's functions and Separation of Variables. Asymptotic methods for the global analysis of ordinary differential equations will be introduced, *e.g.*, boundary layer theory and WKB theory. The Calculus of Variations will enable us to understand the different formulations of mechanics (by Newton, Lagrange and Hamilton). Time permitting, we will finish with an introduction to Fluid Dynamics.

Grading: Your grade will be based on class participation, homework problems and quizzes. The dates for homework and quizzes will be determined by the pace of the class, so attendance is important.

Course Topics (not of equal weight; some is review)

1. Linear Systems $\mathbf{Ax} = \mathbf{b}$ (with \mathbf{A} symmetric positive definite) - Strang Chapter 1. Subtopic: minimum principles
2. Equilibrium of Discrete Systems - Strang Chapter 2. Subtopic: Constraints and Lagrange multipliers.
3. Equilibrium of Continuous Systems - Strang Chapter 3. Subtopic: Sturm-Liouville problems.
4. Eigenfunction expansions for linear ODE boundary value problems and linear algebraic equations.
5. Green's functions for ODEs.
6. Local behavior at irregular singular points of linear ODEs - Bender and Orszag Chapter 3. Subtopic: The method of dominant balance.
7. Introduction to asymptotic methods for ODEs:
 - (a) boundary layer theory - Bender & Orszag Chapter 9
 - (b) WKB theory - Bender & Orszag Chapter 10
 - (c) multiple scales analysis - Bender and Orszag Chapter 11
8. Calculus of Variations: (a) Equilibrium, (b) Dynamics. Subtopic: Different formulations of mechanics.
9. Introduction to Fluids