

Math 703
Methods of Applied Mathematics I
MWF 9:55-10:45 in Van Vleck B203

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Textbook 1: *Introduction to Applied Mathematics*, Strang, Wellesley-Cambridge Press.

Textbook 2: *Advanced Mathematical Methods for Scientists and Engineers*, Bender and Orszag, Springer.

Prerequisites: Elementary Linear Algebra and Elementary Differential Equations.

Course description: The course introduces methods to solve mathematical problems that arise in areas of application such as physics, engineering and statistics. Roughly speaking, we can divide these problems into two categories: (i) *equilibrium, i.e.*, statics problems (ii) and *departures from equilibrium, i.e.*, dynamics problems. The first part of the course will be devoted to the study of equilibrium; linear algebra provides a unifying framework for discrete equilibrium problems from several application areas. This algebraic structure is also the basis for numerical solution of both discrete and continuous equilibrium systems. In the continuous case, equilibrium mechanics leads to boundary value problems for differential equations: in one dimension, one finds ordinary differential equations, *e.g.*, Sturm-Liouville equations; for higher dimensional systems, one finds partial differential equations, *e.g.*, Laplace's equation, Poisson's equation and the equations for Stokes flow. Laplace's equation in turn leads naturally to the study of Complex Variables.

Dynamics problems become initial value problems for ordinary and partial differential equations. Thus we will study some well-known techniques for solving differential equations, *e.g.*, Green's functions and Separation of Variables. Asymptotic methods for the global analysis of ordinary differential equations will be introduced, *e.g.*, boundary layer theory and WKB theory. The Calculus of Variations will enable us to understand the different formulations of mechanics (by Newton, Lagrange and Hamilton). We will finish with an introduction to Fluid Dynamics.

Grading: Your grade will be based on in-class solution of homework problems and a term project related to material covered in class. Your attendance will be mandatory on days devoted to problem solving (selected days announced in advance). Exceptions will be made only for official UW business (travel to conferences, job interviews, etc).

Some suggested topics for projects are:

- Semi-direct and Iterative Methods (Strang 5.3)
- The Finite Element Method (Strang 5.4)
- The Fast Fourier Transform (Strang 5.5)
- Approximate Solution of Difference Equations (B&O Chapter 5)
- Asymptotic Expansion of Integrals (B&O Chapter 6)
- Summation of Series (B&O Chapter 8)
- The Mathieu Equation and Stability (B&O 11.4)
- Network Flows and Combinatorics (Strang Chapter 7)
- Optimization (Strang Chapter 8)

Course Topics

1. Linear Systems $\mathbf{Ax} = \mathbf{b}$ (with \mathbf{A} symmetric positive definite)- Strang Chapter 1. Subtopics: Gaussian elimination, minimum principles.
2. Equilibrium of Discrete Systems- Strang Chapter 2. Subtopic: Constraints and Lagrange multipliers.
3. Equilibrium of Continuous Systems - Strang Chapter 3. Subtopic: Sturm-Liouville problems.
4. Eigenfunction expansions for linear ODE boundary value problems and linear algebraic equations.
5. Green's functions for ODEs.
6. Local behavior at irregular singular points of linear ODEs - Bender and Orszag 3.4. Subtopic: The method of dominant balance.
7. Introduction to asymptotic methods for ODEs, *e.g.*, boundary layer theory and WKB theory - sections of Chs. 7, 9, 10 and possibly 11 in Bender and Orszag.
8. Calculus of Variations: (a) Equilibrium, (b) Dynamics. Subtopic: Different formulations of mechanics.
9. Fluids, Laplace's Equation, Complex Variables.