

You should know the following concepts:

1. Write a sample space and decide if it contains equally likely outcomes
2. Draw a tree model
3. Draw an area model
4. Compute probabilities using either of the above models
5. Compute expected value
6. Decide if two conditions are independent or dependent
7. The various ways the equals sign can be used
8. Possible student misconceptions about the equals sign
9. The various ways letters can be used
10. Finding formulas for patterns and relating them to the picture
11. Relating qualitative graphs to stories
12. Solving proportions problems in multiple ways
13. Writing equations for proportions and graphing them
14. Solving linear problems in multiple ways
15. Writing equations for linear situations and graphing them
16. Recognize what makes a relationship proportional, linear, quadratic, or inverse.
17. Calculate the mean, median, and mode
18. Graph data in histograms, pie charts, and box-and-whisker plots
19. Methods of computing the mean
20. Evaluate studies and graphs for bias and accuracy

Practice Problems

1. You have a 6-sided die, where the six sides are labeled 1, 2, 3, 5, 7, 10.
 - (a) What is the expected value of rolling this die?
 - (b) What is the probability that the sum of two rolls is at least ten?
 - (c) What is the probability that you roll at least one 5 in three rolls?
2. You have two spinners. One is divided into three equal sections, labeled 1, 2, and 3. The other is divided into five equal sections, labeled 2, 3, 5, and 7, 9. You spin both spinners.
 - (a) What is the probability that the product of the spinners is even?
 - (b) What is the probability of getting a 2 on at least one spinner?
 - (c) Are parts a and b independent?

3. You flip a coin three times. You get ten points if all three flips are heads, otherwise you get three points per tails.

- (a) Write at least two sample spaces for this game, and decide if they contain equally likely outcomes
- (b) What is the probability that you score at least six points?
- (c) What is the expected value of playing this game?

4. You are going to select a random house in Madison. Decide if the following conditions are independent:

- (a) I. The house has Christmas lights up. II. The house contains a Christmas tree.
- (b) I. The house contains a Christmas tree. II. The house has a car in the garage.
- (c) I. The house has a car in the garage. II. The house is painted white.

5. Explain how the following uses of the equals sign are different:

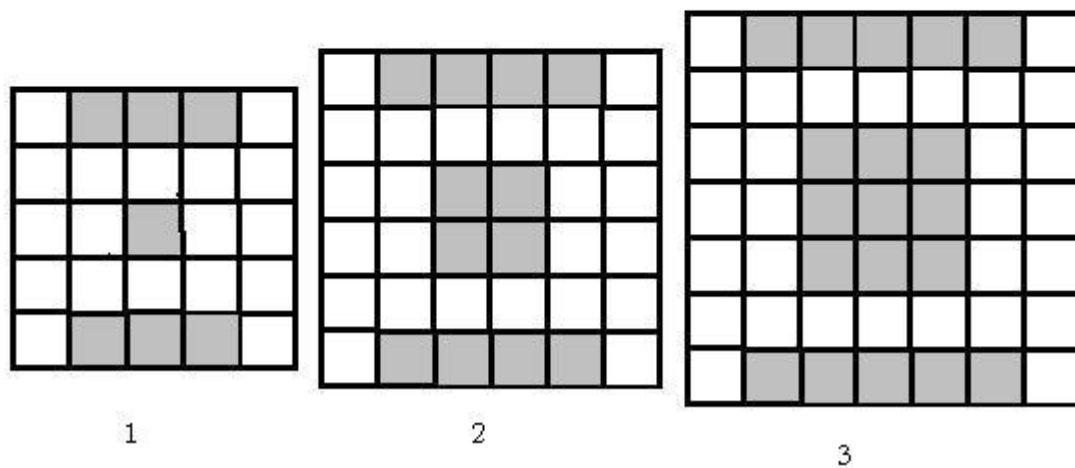
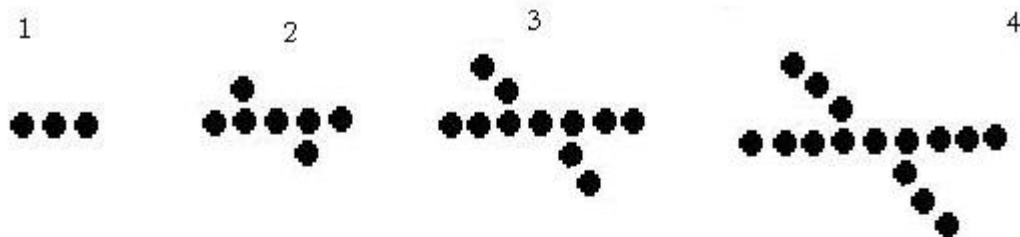
$$x + 3 = x + 2$$

$$y = 5x + 2$$

6. Give two different ways the letter b can be used mathematically.

7. (a) Give two different formulas for the number of circles in the n -th stage, and relate your formulas to the picture.

(b) Give two different formulas for the number of shaded squares in the n -th stage, and relate your formulas to the picture.



8. Solve the following problems in two different ways:

- (a) If I need five buckets of water for eight rosebushes, how many buckets do I need for twelve rosebushes?
- (b) My shopping cart can hold 6 bags of rice or 20 bags of beans. If I have 3 bags of rice, how many bags of beans do I have room for?

9. Graph and write equations for the following situations. Find and interpret the slope, x -intercept, and y -intercept.
- I need four pillowcases for every fitted sheet.
 - Snow is falling at a constant rate. Two hours after it started, there were 5 inches on the ground. Three hours after it started, there were 6.5 inches on the ground.
 - A 5-gallon water jug has a hole in it. Two minutes ago, it contained 4 gallons. One minute ago, it contained 3 gallons.
10. Draw two different graphs that could represent the following story. Make sure you label your axes:
- I walked up Bascom Hill to Van Vleck. I climbed two flights of stairs, rested, then climbed three more flights of stairs. Then I hurried back down the stairs, and ran back down Bascom Hill.
11. One runner starts the course at 5 mph. Fifteen minutes later, another runner starts at 6 mph.
- When and where would the two runners meet?
 - If the race is 15 miles long, how long after the first runner starts should the second one begin so that the race will end in a tie?
12. (a) Find the mean, median, and mode of $\{2, 4, 5, 6, 6, 6, 8, 9, 12, 12, 12, 12\}$
- (b) Represent the numbers graphically three different ways, including a box-and-whisker plot.
13. Are the following conclusions justified?
- In a survey, 98% of Republicans and 96% of Democrats say they brushed their teeth this morning, with a margin of error of 5%. Conclusion: Republicans brush their teeth more often than Democrats.
 - A study randomly assigns 20 people to either be woken up in the middle of the night or not. After a week, the group that was woken up in the middle of the night has 50% more accidents in a driving simulation than the group that was not. Conclusion: Interrupted sleep decreases the safety of your driving.
 - A study replaces half the participants coffee with hot chocolate for a month. At the end of the month, the hot chocolate group has lower cholesterol than the coffee group. Conclusion: Decreasing coffee intake lowers cholesterol.
14. Determine if the following methods of finding a mean are accurate. Explain why or why not (with an example if false)
- To average 10, 10, 15, 20: The smallest number is 10. So I find that 15 is 5 bigger than 10, and 20 is 10 bigger than 10. $10+5=15$. I divide by four, the number of numbers. $15/4=3.75$. Then the average is $10 + 3.75 = 13.75$
 - To average 1, 5, 19, 23: Reduce the four numbers to three by taking the number in between each pair: 3, 12, 21. Reduce the three number to 2 the same way: 7.5, 16.5. Then reduce the two numbers to one the same way: 12. That is the average.
15. Create a list of numbers with a mode of 10 and a mean of 100.

Answers to the problems:

1. (a) $4\frac{2}{3}$
(b) $\frac{17}{36}$
(c) $\frac{61}{108}$
2. (a) $\frac{7}{15}$
(b) $\frac{7}{15}$
(c) No
3. (a) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} - equally likely, {3, 6, 9, 10}
- not equally likely, {3 heads, not 3 heads} - not equally likely
(b) $\frac{5}{8}$
(c) 5.75 points
4. (a) Probably dependent - houses with Christmas trees are more likely to put up Christmas lights than those without Christmas trees
(b) Probably Independent - I think an equal percentage of houses with or without cars in the garage would put up Christmas trees
(c) Probably Independent - I think an equal percentage of houses with or without cars in the garage would be painted white
5. One is false (the set of numbers that make it true when plugged in for x is empty). The other describes a relationship between y and x .
6. As a variable, $5b + 3 = 2b - 4$, or as a parameter, $y = mx + b$, or as part of a unit, $w = 5lb$
7. (a) $2n + 1 + 2(n - 1)$ The horizontal piece, plus two diagonal arms
 $2n + 2(n + 1) - 3$ Two arms of length n , two of length $n + 1$, and we counted the center 4 times so we subtract 3.
(b) $n^2 + 2(n + 2)$. Center square plus two lines
 $(n + 4)^2 - 2(n + 4) - 4(n + 1)$. The whole grid, minus the two side edges, minus the four sides of the white square.
8. (a) seven and a half
(b) ten
9. Graph and write equations for the following situations. Find and interpret the slope, x -intercept, and y -intercept.
 - (a) $p = 4s$. Both intercepts are zero, since this is proportional. Slope is either 4 or $\frac{1}{4}$, depending on which way you graph it.
 - (b) $y = 2 + 1.5x$ Slope is 1.5 inches per hour - represents the rate of snowfall. y -intercept is 2 inches, the snow on the ground before this storm.
 - (c) $y = 2 - x$ Slope is -1 gallons per minute, the rate of water loss. y -intercept is 2 gallons, the water level right now. x -intercept is 2 minutes, the time until it will be empty
- 10.
11. (a) 7.5 miles down the course, an hour and a half after the first runner started.
(b) Half an hour after the first runner.
12. (a) mean 7.83, median 7, mode 12.
(b)

13. (a) No. The margin of error is larger than the reported difference. Also, what people say about brushing their teeth and what they actually did may be very different.
- (b) Yes, this is probably justified.
- (c) No, it could have been the increase in hot chocolate instead! Or perhaps even a difference in the quality of coffee vs hot chocolate offered.
14. Determine if the following methods of finding a mean are accurate. Explain why or why not (with an example if false)
- (a) Yes, this is a valid method. Algebraically, $a + \frac{b-a}{4} + \frac{c-a}{4} + \frac{d-a}{4} = \frac{a+b+c+d}{4}$.
- (b) No. This method gives too much weight to the middle numbers. For example, on 1, 3, 5, 11: We would get 2, 4, 8, then 3, 6, then 4.5. But the correct mean is 5.
15. 10, 10, 280 works. There are many other possibilities.