1 Sets

1. At UW-Madison every English major has to take at least one foreign language course. 170 students were asked about their foreign language course and here is the result:
   75 students take French
   80 students take German
   35 students take French and German
   33 students take German and Russian
   30 students take French only

   (a) How many students take Russian?

   (b) How many students take French and Russian but not German?

2. Circle the correct answer. No reasoning required.

   (a) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$. True / False

   (b) $n(E') = n(E) + 1$. True / False

3. Mrs. Smith takes her 19 first graders to the ice cream shop. The students and Mrs. Smith each choose an ice cream flavor and one mix in. Each of them orders a different combination. There are 6 flavors of ice cream. What is the minimum number of mix ins to make this situation possible?

4. During February, Lee's Motors sold 110 cars with air conditioning, 95 with power steering and 100 with automatic transmissions. 5 cars had all three options, 10 cars had none of these options. 20 cars had only air conditioning, 60 cars had only automatic transmissions and 30 cars had only power steering. 10 cars had both automatic transmission and power steering.

   (A) How many cars were sold in February?

   (B) How many cars had neither power steering nor automatic transmissions?

5. Suppose $n(A \times B) = 24, n(A \cup B) = 10$ and $n(A \cap B) = 1$. Find $n(A)$ and $n(B)$.

6. During February, Lee's Motors sold 75 cars with air conditioning, 95 with power steering and 100 with automatic transmissions. Twenty cars had all three options, 10 cars had none of these options, and 10 cars were sold that had only air conditioning. In addition 50 cars had both automatic transmission and power steering, and 60 cars had both automatic transmissions and air conditioning.

(A) How many cars were sold in February?

(B) How many cars had only power steering?

7. Let A and B be subsets if U with $n(U) = 50$, $n(A' \cap B') = 20$,and $n(A \cap B) = 6$. Find the number of elements which are both in A or in B but not in both.

8. We have the following data from 100 students about how they commute to campus.
   8 drive at least part of the time
   20 use the bus at least part of the time
   48 ride a bicycle at least part of the time
   38 do none of these
   No student whoever drives a car also uses the bus
   How many students who ride a bicycle also drive a car or use the bus?

9. In a survey of 125 college students, it was fount that of three newspapers, the *Wall Street Journal, New York Times* and *Chicago Tribune*: 60 read the *Chicago Tribune*, 40 read the *New York Times*, 15 read the *Wall Street Journal*, 25 read the *Chicago tribune* and *New York Times*, 8 read the *Wall Street Journal* and *New York Times*, 3 read the *Chicago Tribune* and *Wall Street Journal* and 1 read all three.

   (a) How many read non of these papers?

   (b) How many read only the *Chicago Tribune*?

   (c) How many read neither the *Chicago Tribune* nor the *New York Times*?

10. How many ways are there to line up 8 male and 5 female students?

11. A class of 100 students has 60 freshman and 40 sophomores. 45 of the freshmen are Wisconsin residents and 30 of sophomores are Wisconsin residents. Find the probability that a randomly selected student is

    (a) A Wisconsin resident

    (b) A Wisconsin resident given s/he is a sophomore.

    (c) A freshman given s/he is not a Wisconsin resident.

    (d) Are the two events, being a Wisconsin resident and being a sophomore independent or not. Justify your answer.

2 Counting

1. Mrs. Smith takes her 19 first graders to the ice cream shop. The students and Mrs. Smith each choose an ice cream flavor and one mix in. Each of them orders a different combination. There are 6 flavors of ice cream. What is the minimum number of mix ins to make this situation possible?

2. A business professor refers to his students in his Business 101 course by their first and last name initials. For example, John Smith would be called JS. A mathematics professor points out to him that there are at least two students in the course with the same initials. How many students are in Business 101?

3. How many different 9-letter "words" are there made out of the letters in COMMITTEE?

4. How many different 7-letter "words" are there made out of the letters in MINIMUM?

5. We have a group of 7 men and 6 women to line up.

   (A) How many line ups are possible?

   (B) How many line ups are possible if men and women are to be alternated?

   (C) How many line ups are possible if men and women are to be in blocks and separate from each other?

   (D) How many line ups are possible if there are 3 men first then 2 women followed by 2 men, 3 women, 2 men and 1 women?

6. The student union board has 12 members, 7 males including John and 5 females including Samantha. Four tasks are to be assigned including that of reserving a room for meetings. The tasks are assigned at random, at most one task per person.

   (A) Find the probability that John is given a task.

   (B) Find the probability that John is asked to reserve a room.

   (C) Find the probability that both men and women are given tasks.

   (D) Find the probability that neither John nor Samantha is assigned a task?

   There are 7 boys including Sam, 6 girls including Jane and 5 teachers including Mrs. Smith and Mr. Lee who are willing to serve on a committee. There are 9 people on the committee consisting of 3 boys, 4 girls and 2 teachers.

   (A) What are the chances that both Sam and Jane are on the committee?

(B) What are the chances that neither Sam and Jan are on the committee but both Mrs. Smith and Mr. Lee serve?

(C) Sam and Jane do not get along. What are the chances that just one of them is on the committee?

(D) What are the chances that exactly one of Sam and Mrs. Smith serve and neither Jane nor Mr. Lee serve on the committee?

7. A ceremony is to include 7 speeches and 6 musical selections.

   (A) How many programs are possible?

   (B) How many programs are possible if speeches and musical selections are to be alternated?

   (C) How many programs are possible if speeches and musical selections are to be in blocks and separate from each other?

   (D) How many programs are possible if there are 3 speeches first then 2 musical selections followed by 2 speeches, 3 musical selections, 2 speeches and 1 musical selection?

8. There are 25 cans of cola in a cooler. 9 are regular and the remainder are diet. In how many ways can you select each of the following?

   (A) 4 can from the cooler

   (B) 4 regular colas

   (C) 4 colas of which at least 2 are regular

   (D) 4 colas of which there are both regular and diet colas

3 Probability

1. The student union board has 12 members, 7 males including John and 5 females including Samantha. Four tasks are to be assigned including that of reserving a room for meetings. The tasks are assigned at random, at most one task per person.

   (A) Find the probability that John is given a task.

   (B) Find the probability that John is asked to reserve a room.

   (C) Find the probability that both men and women are given tasks.

   (D) Find the probability that neither John nor Samantha is assigned a task?

There are 7 boys including Sam, 6 girls including Jane and 5 teachers including Mrs. Smith and Mr. Lee who are willing to serve on a committee. There are 9 people on the committee consisting of 3 boys, 4 girls and 2 teachers.

(A) What are the chances that both Sam and Jane are on the committee?

(B) What are the chances that neither Sam and Jan are on the committee but both Mrs. Smith and Mr. Lee serve?

(C) Sam and Jane do not get along. What are the chances that just one of them is on the committee?

(D) What are the chances that exactly one of Sam and Mrs. Smith serve and neither Jane nor Mr. Lee serve on the committee?

2. What is the probability that out of nine students exactly 2 have the same birthday month and 3 have the same birthday month but different than the other two and the rest do not have their birthday months same as anybody else in the group?

3. Three 5 sided dice, green, red and yellow are rolled and numbers on the tops are noted. Numbers on the sides are 1,2,3, 4 and 5.

   (A) What are the chances that all three numbers are the same?

   (B) What are the chances that the red and green dice have the same number?

   (C) What are the chances that exactly two dice have the same number?

   (D) What are the chances that the sum of three numbers is 4?

4. Suppose events E and F are mutually exclusive and p(E)=0.8 and p(F)=0.1. Find p($E \cup F$).

5. Suppose events E and F are independent and p(E)=0.8 and p(F)=0.1. Find p($E \cup F$).

6. There are two urns, A and B. Urn A contains 5 blue and 6 white balls. Urn B contains 4 blue, 2 white, and 3 yellow balls. An urn is chosen at random and then a ball is chosen from that urn and its color is noted.

   (A) Draw the tree for this experiment with probabilities filled in.

   (B) Find the probability that a white ball was chosen.

   (C) Find the probability that urn A was picked given that a white ball was chosen.

   (D) Find the probability that urn A was picked given that neither a blue nor a white ball was chosen.

7. A fair coin is flipped until there are a total of 2 heads or a total of 5 flips. A random variable **X** is defined as the number of flips.

   (A) What are the possible values for **X**?

   (B) Find the density function for **X**

   (C) Calculate the expected value, variance, and standard deviation of **X**

8. A professor (who shall not be named) intends to bring her briefcase to the office each morning. She forgets it one-quarter of the time. Assume that forgetting the briefcase is a Bernoulli trial. Find the probability that she forgets it at least twice a week (5 days).

9. During the season a basketball player makes 100 attempts at 3-point shots with %40 success rate and 400 attempts at 2-point shots with %30 failure rate. How many points do you expect him to win for his team?

10. You pay $5 to play the following game. Three dice, green, red and yellow are rolled and numbers on the tops are noted. If the three numbers are the same you win $15, if exactly two of them are the same you win $8, otherwise you lose. Find the expected value of the winning/loss for this game.

11. An aquarium contains 6 fish: 4 fish weigh 200 grams and 2 fish weigh 100 grams. 2 fish are selected at random. A random variable **X** is defined by the total weight of the 2 fish.

    (A) What are the possible values for **X**?

    (B) Find the density function for **X**

    (C) Calculate the expected value, variance, and standard deviation of **X**

12. Two fair dice are rolled, and the result on each die is noted. Find the probability that

    (a) The sum is 8, given at least one of the numbers is even.

    (b) At least one of the numbers is even, given that the sum is 8.

    (c) At least one of the number is even, given the sum is 5.

13. On an exam there are 10 multiple choice questions, with 5 options and each with exactly one correct answer. What is the probability that

    (a) One student gets exactly 6 correct answers just by guessing.

(b) Two students get exactly 6 correct answers just by guessing, assuming their answers are independent of each other.

(c) If a student guesses the answers at random, how many questions do you expect him to get right?

Suppose the average height of 1000 senior male students is 180 cm and the standard deviation is 6 cm. Suppose also that the students' heights closely follow a normal distribution.

(a) How many of these students would you expect to be taller than 190 cm?

(b) How many 175 cm or shorter?

(c) How many at least 177 cm and at most 186 cm tall?

14. A random variable X has the density function shown below. Find the missing entries in the table.

Value of X	Probability	Product
?	?	?
1	.2	.2
?	.12	.6
3	?	.3
4	?	1.2
		E[X]=2.1

15. An examination consists of 3 true-false and and 4 multiple choice questions (5 options per question), each with exactly 1 correct answer. If a student selects answers at random, one answer per question, what is the expected number of correct answers.

16. What is the probability that 10 students have exactly the same birthday month?

17. Mary has 2 dime and a nickel in her right-hand pocket and 1 dime and and a nickel in her left-hand pocket. She selects a coin at random from her right-hand pocket and puts it in her left-hand pocket. Then he selects a coin at random from her left-hand pocket. A random variable X is defined as the total number of dimes selected. Find the expected value of X.

18. There are 8 mice in a cage: 3 white males, 3 gray females and 2 gray males. Two mice are selected simultaneously and at random and their colors is noted.

(A) Find the probability that at least one mouse is a male, given that exactly one is gray.

(B) Find the probability that at least one mouse is a male, given that both are gray.

(C) Find the probability that at least one mouse is gray if one is male and the other is female.

19. A box contains 40 red balls and 60 white balls. 10 balls are selected one after the other with replacement and their color is noted. What is the probability that there are at least 7 red balls chosen?

20. A poker hand consists of 5 cards selected at random from an ordinary deck. Find the probability that a poker hand contains:

    (A) 4 kings

    (B) 4 of a kind (4 cards of the same rank)

    (C) 5 consecutive cards of the same suit (each ace can be used either as the first card, A2345,or as the last card, 10JQKA.

21. Shirin, Ashley, Adrian, Leah, John and Danny are friends. Find the probability that

    (A) they all have the same birthday.

    (B) Leah and John have the same birthday and everybody else has a different birthday.

    (C) Exactly 2 of them have the same birthday and the rest have different birthdays.

22. A major league baseball player has a batting average (number of hits divided by number of times at bat) of 0.2. Suppose the player's turns at bat are viewed as Bernoulli trials with $p = .2$. Let the random variable X be the number of hits.

    (A) Calculate the mean and standard deviation for X for 200 times at bat.

    (B) What is the probability that in 200 times at bat he has at least 50 hits?

4 Linear Models

1. A water reservoir loses water at a constant rate. On the 7th day it has 885.16 million gallons of water and on the 12th day it contains 874.56 million gallons of water. Write a linear equation describing the situation and find when the reservoir will be empty.

2. (a) Find the equation of the line going through points $(-1, -1)$ and $(-1, 1)$.

   (b) Find the equation of the line parallel to $2x + 4y = 5$ and y-intercept 3.

(c) Find the y-intercept and x-intercept of $3x + 2y = 4$.

3. We need 5 cups of flower and 2 cups of sugar to make a lemon cake. We need 3 cups of flower and 1 cup of sugar and 2 bananas for banana bread. We have a total of 137 cups of flower and 54 cups of sugar. How many bananas do we need if we are to make both pastries and use up all the ingredients?

4. Find the feasible set defined by $2x + y \geq 4$, $x + 2y \geq 4$, $x \geq 0$ and $y \geq 0$. Maximize the objective function $x + y$ on the found feasible set.

5. A coin collection consists of 37 coins, nickels, dimes and quarters. If the collection has the face value of \$3.25 and there are 5 more dimes than there are nickels, how many of each coin are in the collection?

6. Find the feasible set defined by $2x + y \geq 6$, $x + 2y \geq 4$, $x \geq 0$ and $y \geq 0$. Maximize the objective function $x + y$ on the found feasible set.

7. A theater has 500 seats, divided into orchestra, main and balcony seating. Orchestra seats sell for \$75, main seats for \$50 and balcony seats for \$35. If all the seats are sold, the gross revenue is $\$23,000$. If all the main and balcony, but only half the orchestra seats are sold, the gross revenue is $\$21,500$. How many are there of each kind of seat?

8. Find the feasible set defined by $2x + y \leq 4$, $x + 2y \leq 4$, $x \geq 0$ and $y \geq 0$. Maximize the objective function $x + y$ on the found feasible set.

9. An amount of \$6500 isnplaced in three investments at rates of %6, %8 and %9 per annum, respectively. The total annual income is \$480. If the income from the third investment is \$60 more than the income from the second investment, find the amount of each investment.

10. Find the feasible set defined by $4x + y \geq 4$, $3x + 5y \leq 15$, $x \geq 0$ and $y \geq 2$. Maximize the objective function $x + y$ on the found feasible set.

11. At a movie theater an adult ticket costs \$7.50 and a child's ticket costs \$5.25. In one afternoon the theater sells 350 tickets and makes \$2400. How many adults and how many children were at the movies that afternoon?

12. A ski manufacturer makes two types of skis: downhill and cross-country. Using the information given below, how many of each type of ski should be made for a maximum profit to be achieved? Define your variables carefully and set up the problem. Do not graph and solve the problem, though.

	Downhill	Cross-country	Maximum time available
Manufacturing time	2 hours	1 hour	40 hours
Finishing time per ski	1 hour	1 hour	32 hours
Profit per ski	\$70	\$50	

13. A bakery makes two types of cakes each day: poppy seed and German chocolate. The profit to the bakery is \$3 on each poppy seed cake and \$4 on each German Chocolate cake. A poppy seed cake requires 450 grams of flour, 200 grams of butter and 110 grams of poppy seeds. A German chocolate cake requires 650 grams of flour, 120 grams of butter and 150 grams of chocolate. There 9800 grams of flour, 2500 grams of butter, 1500 grams of poppy seeds and 2000 grams of chocolate. the objective is to maximize profit. Carefully define variables and set up the linear programming problem. DO NOT SOLVE IT

14. At an ice cream factory, they make two types of ice cream, regular and low-calorie. Each gallon of regular ice cream requires .5 gallon of milk, 1.1 pounds of sugar and .6 gallon of cream. Each gallon of low-calorie ice cream requires .7 gallon of milk, .9 pounds of sugar and .4 gallon of cream. Each day we have 900 gallons of milk, 500 pounds of sugar and 350 gallons of cream. We also know that at least half of all the ice cream production must be of the regular kind. The profit per gallon is \$ 1.20 for regular ice cream and \$ 1.50 for low-calorie ice cream.
Define your variable carefully and set up the problem. You do not need to solve it.

5 Matrices and solving systems

1. Circle the correct answer.No reasoning required.

   (a) The equation $x_1 + x_2 = 1$ has
   a)no solution b)1 solution c)2 solutions d)infinitely many solutions.

   (b) The system $x_1 + x_2 = 1$ and $x_1 - x_2 = -1$ has
   a)no solution b)1 solution c)2 solutions d)infinitely many solutions.

(c) The system $x_1 + x_2 = 1$ and $2x_1 + 2x_2 = 1$ has
a)no solution b)1 solution c)2 solutions d)infinitely many solutions.

(d) If A is a 2×2 invertible matrix, then the matrix equation $A\mathbf{x} = \mathbf{0}$ has
a)no solution b)1 solution c)2 solutions d)infinitely many solutions.

(e) $(A^{-1}BC^{-1})^{-1} =$
a)$AB^{-1}C$ b)$B^{-1}AC$ c)CAB^{-1} d)$CB^{-1}A$.

2. Consider

$$\begin{cases} 2x_1 + 3x_2 &= 8 \\ 3x_1 - x_2 &= 1 \end{cases}.$$

(a) Rewrite the above system in matrix notation $A\mathbf{x} = b$.

(b) Solve the $A\mathbf{x} = b$ system using inverse matrix if possible.

3. Decide if the following matrix is invertible, if yes find its inverse.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Find x_1, x_2, x_3 so that the following equation holds.

$$\begin{pmatrix} x_1 + x_2 - x_3 & 0 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & x_1 - x_2 - x_3 \end{pmatrix}$$

5. Solve the following system:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + 2x_2 + x_3 + x_4 &= 0 \end{cases}$$

6. **(A)** Find the inverse of matrix A, if possible.

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$$

(B) Find a 2×2 matrix B, such that $AB = C$.

$$C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

7. Solve the following system:

$$\begin{cases} x_1 - x_2 + x_3 - x_4 &= 0 \\ 2x_1 - 2x_2 + x_3 - x_4 &= 0 \end{cases}$$

8. **(A)** Find the inverse of matrix A, if possible.

$$A = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix}$$

(B) Find a 2×2 matrix B, such that $AB = C$.

$$C = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

9. Solve the following system:

$$\begin{cases} 2x_1 + x_2 - x_3 + 5x_4 &= 0 \\ 3x_1 + 2x_2 + x_3 - 2x_4 &= 0 \end{cases}$$

10. For what values of c the matrix AB does not have an inverse

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & c \end{bmatrix}$$

11. Solve the following system:

$$\begin{cases} x + y + z &= 10 \\ 2x - y - z &= 5 \\ -3x + y - 4z &= -20 \\ x + 5z &= 15 \end{cases}$$

12. **(A)** Find the inverse of matrix A, if possible.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix}$$

(B) Find a 3×1 matrix B, such that $AB = C$ (Matrix A from previous section).

$$C = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

6 Markov Chains

1. A rental car company opens two offices, East and West with 50 cars at each location. Every morning each car is rented and returned at the end of the day. %60 of the cars rented at East and %20 of the cars rented at West are returned to the East office. The remaining cars are returned to the West location.

   (A) Find the transition matrix for this problem.

   (B) In the long run, how will these 100 cars be distributed among the two offices.

   (C) If a car is at the East office today, how many days would you expect it to be at the West office before it is returned to the East location?

2. A rental car company opens two offices, East and West with 50 cars at each location. Every morning each car is rented and returned at the end of the day. %40 of the cars rented at East and %80 of the cars rented at West are returned to the East office. The remaining cars are returned to the West location.

   (A) Find the transition matrix for this problem.

   (B) In the long run, how will these cars be distributed among the two offices.

   (C) If a car is at the West office today, how many days would you expect it to be at the East office before it is returned to the West location?

3. A Markov chain with 3 states has transition matrix $\mathbf{P}$. In words describe what $\mathbf{P_{32}(4)}$ stands for.

4. A Markov chain with 3 states has transition matrix $\mathbf{P}$. In words describe what $\mathbf{P_{23}(4)}$ stands for.

5. Mike has \$1 and John has \$2. They flip a coin with p(H)=$\frac{2}{3}$. If they get a head, Mike pays John \$1, otherwise John pays Mike \$1. The game ends when one of them has ran out of money and the other one has \$3.

   (A) Find the transition matrix for this problem fromthe poit of view of Mike.

   (B) How long would you expect this game to last?

   (C) What is the probability of Mike winning the game?

6. Stephen has \$2 and Hanjun has \$4. They flip a coin with p(H)=$\frac{2}{3}$. If they get a head, Stephen pays Hanjun \$2, otherwise Hanjun pays Stephen \$2. The game ends when one of them has ran out of money and the other one has \$6.

(A) Find the transition matrix for this problem.

(B) How long would you expect this game to last?

(C) What is the probability of Stephen winning the game?

7. A city of 600,000 population has a one grocery store, Copps. A new grocery store, Woodman's, opens on July 1st. It is known that if somebody shops at Copps one month, there is 5% that s/he will be shopping at Woodman's the next month and if somebody shops at Woodman's one month, there is 10% that s/he will be shopping at Copps the next month.

   (A) Find the transition matrix for this problem.

   (B) How many people will be shopping at each store in August?

   (C) In the long run how many people will be shopping at each store?

8. A Markov Chain has the transition matrix

$$\begin{bmatrix} .6 & .4 \\ .2 & .8 \end{bmatrix}.$$

   Find the probability of going from state 2 to state 1 after 2 stages (transitions).

9. A small animal lives in a territory that can be divided into two ares described as meadow and woods and it moves randomly from one area to another. If it is in the woods on one observation, then it is twice as likely to be in the woods as meadow on the next observation. If it is in the meadow on one observation, then it is three times as likely to be in the meadow as woods on the next observation.

   (a) Find the transition matrix.

   (b) If the animal is in the woods on the first observation, find the probability that it is in the meadow on the third observation. (Do NOT use a tree diagram.)

10. A not-so-enthusiastic students attends class in the following manner. If he is absent one day he will be absent the next day with the probability of .3 and if he is present one day he will be absent the next day with the probability of .6.

   (a) Find the transition matrix.

   (b) How often in the long run does our not-so enthusiastic student attend class. (Consider the stable probability vector.)

11.)Decide which of the following transition matrices is regular(if any).

(a) $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ 1 & 0 \end{bmatrix}$

7 Financial

1. John takes a $\$10,000$ car loan to be repaid by $\$250$ monthly payments over 5 years. How much interest does he end up paying the bank?

2. We would like to make quarterly withdrawals of $\$6,000$ from a savings account with annual interest rate of 3% (compunded quarlterly) for 5 years. At the end of 5 years we would like to have $5000 left in the account. How much money should be in the account at the beginning of year 1 to make this possible?

3. Suppose we have a $\$1,000,000$ loan payable in 10 years. We have $\$100,000$ to invest in a savings account with annual interest rate of 6% compounded monthly to be used as part of the loan repayment. How much should we put away each month in this savings account in order to have $\$1,000,000$ in 10 years?

4. We take out a mortgage of $\$500,000$ with annual interest rate of $r = .12$ compounded monthly for 30 years.

   (A) How much are the monthly payments?

   (B) How much of the loan is paid off after 15 years?

   (C) Of the first payment how much is principal and how much is interest?

5. We would like to make 4 annual withdrawals of $\$12,000$ from a college fund with annual interest rate of %2.

   (A) How much money should be in the account at the beginning of year 1 to make this possible?

   (B) Now, in addition to the withdrawals described above, suppose we also like to have $1000 left in the account at the end of 4 years for a graduation gift. How much money should be in the account at the beginning of year 1 to make this possible (for both withdrawals and graduation gift)(interest is compounded annually)?

(C) We have 10 years to save money for the amount needed in part (B). We will save the money in an account with annual interest rate of %5 compounded monthly. How much should we save each month?

6. We would like to make 4 annual withdrawals of $\$15,000$ from a college fund with annual interest rate of %3.

   (A) How much money should be in the account at the beginning of year 1 to make this possible?

   (B) Now, in addition to the withdrawals described above, suppose we also like to have \$2000 left in the account at the end of 4 years for a graduation gift. How much money should be in the account at the beginning of year 1 to make this possible (for both withdrawals and graduation gift)(interest is compounded annually)?

   (C) We have 10 years to save money for the amount needed in part (B). We will save the money in an account with annual interest rate of %6 compounded monthly. How much should we save each month?