(1) Practice double and triple integrals. There are many in the book, choose a bunch from homework and from those that were not assigned for homework. You can get more from any Calculus textbook. I am not including examples here because there are too many... Be sure you are good at integrating.

(2) The **moment of inertia** of a plate $S$ about a line $L$ is given by the equation

$$I_L = \int \int_S \delta^2(x, y) f(x, y) dxdy$$

where $\delta(x, y)$ is the perpendicular distance from the point $(x, y)$ to the line $L$, and $f$ is the density function of the plate. (Inertia makes the world go around, not money.) If $L$ is the $x$-axis, or the $y$-axis, then we denote $I_L$ by $I_x$ and $I_y$, respectively. In this case, $\delta_x = y$ and $\delta_y = x$, of course.

Compute the moments of inertia $I_x$ and $I_y$ of the following plate: the region $S$ is bounded by the curves

$$xy = 1, \quad xy = 2, \quad x = 2y, \quad y = 2x \quad x > 0, y > 0$$

and its density is $f(x, y) = 1$.

(3) Explain why iterated integration of a non-negative continuous function over a rectangle describes the volume under the graph of the function.

(4) Describe the geometric interpretation of the change of variable formula in two dimensions.

(5) Give a geometric description of the change of variable formula in the case of spherical coordinates.

(6) Use an appropriate modification of spherical coordinates to calculate the following integral

$$\int \int \int_S \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right) dxdydz$$

where $S$ is the solid bounded by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 

(7) Find the volume of the solid bounded by the $xy$-plane, the cylinder $x^2 + y^2 = 2x$, and the cone $z = \sqrt{x^2 + y^2}$.

(8) Transform the following integral into an iterating integral in polar coordinates.

$$\int_0^2 \left[ \int_x^{\sqrt{3x}} f(\sqrt{x^2 + y^2}) dy \right] dx.$$ 

(9) Consider the mapping defined by the equations

$$x = u + v, \quad y = v - u^2.$$ 

- Compute the Jacobian determinant $J(u, v)$. 

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- PRACTICE PROBLEMS - FIRST MIDTERM MATH 376 -

SPRING SEMESTER 2012
• A triangle $T$ in the $uv$-plane has vertices $(0, 0), (2, 0), (0, 2)$. Describe, including a drawing, its image $S$ in the $xy$-plane.
• Calculate the area of $S$ using a double integral over $S$ and also using a double integral over $T$.

(10) Establish the given equation by introducing a suitable change of variables.

$$\int \int_S f(x + y)dxdy = 2 \int_{-1}^{1} f(u)du$$

where $S = \{(x, y); |x| + |y| \leq 1\}$.

(11) (TRUE-FALSE) (If true, prove the statement. If false, give a counterexample.)

(a) If a function $f$ is bounded over a region $S \subset \mathbb{R}^2$, then the function is integrable.

(b) If a function $f(x, y)$ is integrable on $S$, then the integral in one of its variables only (say $x$) always exists.

(c) Assume we have two changes of variable $(x, y) = (X(u, v), Y(u, v)) = r(u, v)$ and $(u, v) = (U(s, t), V(s, t)) = s(s, t)$. Thus, we can also write $(x, y) = (\hat{X}(s, t), \hat{Y}(s, t)) = r(s(s, t))$. Assume $J_{u,v}(s, t) = 2$ and $J_{x,y}(s, t) = \frac{1}{2}$. Then $J_{x,y}(u, v) = 1$.

(d) A function that is not continuous can sometimes be integrated over open regions of $\mathbb{R}^2$.

(e) If two regions $A$ and $B$ are symmetric with respect to the $x$ axis, then the centroid of $A \cup B$ lies on the $x$-axis.