Be sure to review all problems that were done in the homework.

(1) Practice integrals. There are many in the book, choose a bunch, for example:
- **line integrals**, page 328 # 4, 9, page 331 # 3, 14 and similar ones
- **surface integrals** page 429 # 2, 6, 7, page 436 # 3, 8, 9 and similar ones

Choose more from those sections if you need to practice more. Be sure at each step to differentiate these integrals from the integrals we did last midterm. They will all appear mixed in the exam once we enter the main Theorems (Stokes, etc) and you will have to decide which one is which to solve them.

(2) A force field is given in polar coordinates by

\[ F(r, \theta) = (-4 \sin \theta, 4 \sin \theta) \]

Compute the work done in moving a particle from the point (1, 0) to the origin along the spiral whose polar equation is \( r = e^{-\theta} \). Choose a segment between the two points and calculate the work along it. Is \( F \) conservative? Explain.

(3) Determine whether or not \( f \) is a gradient and, if so, find the potential.

\[ f(x, y) = (\sin(xy) + xy \cos(xy))(1 + x^2 \cos(xy))j. \]

(4) A fluid flows in the \( xy \)-plane, each particle moving radially (along the radius) away from the origin. If the particle is at a distance \( r \) from the origin \( (r = ||x|| \text{ if the particle is at a point } x) \), the speed is \( ar^n \), for some constants \( a \) and \( n \). Determine the values of \( a \) and \( n \) for which the velocity vector field is the gradient of some potential, and find the potential whenever it exists.

(5) Let \( u \) and \( v \) be \( C^2 \) functions on an open connected set \( S \) of the plane. Let \( R \) be a region in \( S \), bounded by a piecewise smooth closed Jordan curve \( C \). Show that

\[ \oint_C uv dx + uv dy = \int \int_R \left(v \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right) + u \left( \frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} \right) \right) dxdy, \]

where \( \oint \) symbolized the line integral for which \( C \) is traversed counterclockwise.

(6) Determine whether or not the following parametrization defines a smooth parametrized surface.

\[ r(u, v) = u \cos v i + u \sin v j + \frac{1}{2} u^2 \sin 2v k. \]

be sure to give the values of the parameters for which the smooth surface is well-defined.
(7) Use Stoke’s Theorem to find the value of the line integral. In each case be sure to describe in which direction you are traversing the curve.

\[ \oint_C (y - z)dx + (z - x)dy + (x - y)dz, \]

where \( C \) is the intersection of the cylinder \( x^2 + y^2 = a^2 \) and the plane \( \frac{x}{a} + \frac{z}{b} = 1, \ a > 0, \ b > 0. \) (You might want to practice this more, they are on page 442).

(8) Use Stoke’s Theorem to calculate the following integral by transforming it into a line integral.

\[ \int \int_S (\text{curl} F) \cdot \mathbf{n} dS \]

where \( F(x, y, z) = xzi - yj + 2y k \) and where \( S \) consists of the three faces not on the \( xz \)-plane of the tetrahedron bounded by the three coordinate planes and the plane \( 3x + y + 3z = 6. \) The normal \( n \) is chosen to be the one pointing out of the tetrahedron.

(9) Let \( S \) be a parametrized surface described by the graph of \( z = f(x, y) \), where \((x, y) \in T \subset \mathbb{R}^2\), the projection of \( S \) on the \( xy \)-plane. Let \( \phi \) be a scalar field. Show that

\[ \int \int_S \phi(x, y, z) dS = \int \int_T \phi(x, y, f(x, y)) \sqrt{1 + \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \ dx \ dy. \]

\[ \int \int_S \phi(x, y, z) dy \wedge dz = - \int \int_T \phi(x, y, f(x, y)) \frac{\partial f}{\partial x} \ dx \ dy. \]

\[ \int \int_S \phi(x, y, z) dz \wedge dx = - \int \int_T \phi(x, y, f(x, y)) \frac{\partial f}{\partial y} \ dx \ dy. \]

(10) Assume \( S \) is a closed surface which bounds a solid \( V \) of the type described in Gauss Theorem. Let \( n \) be the unit outer normal to the surface (which property of \( S \) guarantees the existence of \( n \)?). Define \( \frac{\partial f}{\partial n} = \nabla f \cdot n. \) Prove that

\[ \int \int_S \frac{\partial f}{\partial n} = \nabla f \cdot n dS = \int \int \int_V \nabla^2 f \ dx \ dy \ dz \]

where \( \nabla^2 f \) is the Laplacian of the function \( f \).

(11) (TRUE-FALSE)

- If a vector field defined on a connected region of the plane satisfies the necessary condition to be a gradient, then the vector field is a gradient.
- If a vector field is \( C^1 \) in an open convex region \( S \) of \( \mathbb{R}^3 \), then \( F \) is a gradient if, and only if, \( \text{curl} F = 0 \) on \( S \).
- Let \( y_1 \) and \( y_2 \) be two different solutions of an equation of the form

\[ y'' + ay' + by = 0 \]

where \( a, b \in \mathbb{R}. \) If \( y_1(0) = y_2(0), \) then \( y_1(t) = y_2(t) \) for all \( t \).
- Let \( y \) be a solution of a differential equation of the form

\[ y'' + ay' + by = 0 \]

where \( a, b \in \mathbb{R}. \) If the solutions of the characteristic equation \( r^2 + ar + b = 0 \) are imaginary, then \( \lim_{t \to \infty} y(t) = 0. \)
- For some values of \( a \) and \( b, \) all solutions of the differential equation

\[ y'' + ay' + by = 0 \]
will go to 0 as $t \to \infty$. 