1. Assume \( f : \mathbb{R}^2 \to \mathbb{R} \) is \( C^2 \) on \( \mathbb{R}^2 \). Define \( u(r, \theta) = f(r \cos \theta, r \sin \theta) \). If \( f \) satisfies Laplace equation, that is, if
\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0
\]
prove that for any \( r \neq 0 \) one has
\[
\frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} = 0.
\]

2. Let \( u : \mathbb{R} \to [0, \infty) \) be differentiable. Show that for each \((x, y, z) \neq (0, 0, 0)\)
\[
F(x, y, z) = u(\sqrt{x^2 + y^2 + z^2})
\]
satisfies
\[
\sqrt{\left( \frac{\partial F}{\partial x} \right)^2 + \left( \frac{\partial F}{\partial y} \right)^2 + \left( \frac{\partial F}{\partial z} \right)^2} = \| u'(\sqrt{x^2 + y^2 + z^2}) \|.
\]

3. Find a constant \( c \) such that at any point of intersection of the two spheres
\[
(x - c)^2 + y^2 + z^2 = 3, \quad x^2 + (y - 1)^2 + z^2 = 1
\]
the corresponding tangent planes will be perpendicular to each other.
4. The equation
\[ \sin(x + y) + \sin(y + z) = 1 \]
defines \( z \) implicitly as a function of \( x \) and \( y \). Find \( \frac{\partial^2 z}{\partial x \partial y} \) as a function of \( x, y \) and \( z \). Find \( \frac{\partial^2 z}{\partial x \partial y}(0,0) \) for \( 0 \leq z \leq \pi \).

5. Find the equation of the tangent plane to \( z = f(x, y) \) at \( c \).
   
   (a) \( f(x, y) = x^3 \sin y \), \( c = (0, 0, 0) \).
   
   (b) \( f(x, y) = x^3 y - y^3 x \), \( c = (1, 1, 0) \).

6. Let \( \mathcal{H} \) be the hyperboloid given by \( x^2 + y^2 - z^2 = 1 \).
   
   (a) Prove that at every point \( (a, b, c) \in \mathcal{H} \), \( \mathcal{H} \) has a tangent plane whose normal is given by \( (-a, -b, c) \).
   
   (b) Find an equation for each plane tangent to \( \mathcal{H} \) that is perpendicular to the \( xy \) plane.
   
   (c) Find an equation of each plane tangent to \( \mathcal{H} \) that is parallel to the plane \( x + y - z = 1 \).

7. When \( u \) is eliminated from the two equations \( x = u + v \) and \( y = uv^2 \), we get an equation of the form \( F(x, y, v) = 0 \) which defines \( v \) implicitly as a function of \( x \) and \( y \), say \( v = h(x, y) \). Show that
\[
\frac{\partial h}{\partial x} = \frac{h(x, y)}{3h(x, y) - 2x}.
\]

Find a similar formula for \( \frac{\partial h}{\partial y} \).

Section 9.7 in the book has this and other worked out examples. You can also try #1-6 and #12 in page 302.