

Sample Problems for the 2nd Midterm. These are problems from 222 midterms given in the past. Old exams are available on the web, but be warned that the syllabus changes every year, so that problems which were on the 2nd midterm 2 years ago may show up on this year's 1st midterm. Don't prepare for last year's midterm!

Look for similar problems in the problem section in the notes. Do not assume that the exam will be exactly like these problems. This is just a sample of what the level will be. The length of the problems does not necessarily correspond to the length of the problems in the exam.

The exam will have a True/False question in the spirit of those appearing in pre-lecture quizzes. They are meant to test understanding of the general theory. Feel free to go back to the quizzes and look through them to practice! The question in the exam will ask for a detailed explanation, so be sure to include those in your practice.

Finally, try hard to write the answers correctly as we will penalize incorrect writing. Be sure to include equal signs whenever two expressions are equal, and to avoid them when they are not. Write limits when you are using them, etc. If it is not on the paper, do not assume we will guess what you meant, we will not. Best of luck to all of you!!

- (1) (a) Find

$$\Re \left\{ \frac{e^{(1-i)x}}{1+2i} \right\}.$$

- (b) You are given an angle x whose Cosine and Sine are given by

$$\sin x = A, \text{ and } \cos x = \sqrt{1 - A^2}.$$

Compute $\cos 6x$.

- (2) Solve the following integral using complex numbers

$$\int \sin^2 3x \cos 5x dx$$

- (3) Find and draw all the solutions to the equation

$$z^3 + 29 = 0.$$

- (4) (a) Find the general solution of $\frac{dy}{dx} = (\cos y)^2(1+x)$.

- (b) Which solution satisfies $y(0) = \frac{\pi}{3}$?

- (5) (a) Find the solution of
$$\begin{cases} x \frac{dy}{dx} + 3y = 2 + x^2 \\ y(1) = B \end{cases}$$

- (b) For which value of B does the limit $\lim_{x \rightarrow 0} y(x)$ exist?

- (6) A 100 gallon tank contains originally a saline solution with 60% salt. We start pumping in 5 gallons per minute of pure water. The liquid in the tank is kept well-mixed as we remove 5 gallons per minute of solution to maintain the volume of liquid in the gallon. Find a formula for the percentage of salt in the tank solution at any time t . How long will it take for the solution to half its percentage of salt? (In this problem assume salt is a liquid.) Will the liquid be salt-free for any time t ? How much salt will we have as $t \rightarrow \infty$?

- (7) Find the general solution of $5 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 5y = 0$. For which solution(s) $y(x)$ of the equation does the limit $\lim_{x \rightarrow \infty} y(x)$ exist?

- (8) Find a particular solution of the equation

$$\frac{d^3y}{dx^3} - 5 \frac{dy^2}{dx^2} + \frac{dy}{dx} + 3y = x + \sin x.$$

- (9) The equation for the displacement $y(t)$ of a spring subject to a forced vibration of frequency ω is

$$\frac{dy^2}{dx^2} + 2\frac{dy}{dx} + 2y = \sin(\omega t).$$

Are we considering friction? Find the solution for $\omega \neq 1$ and describe what the behaviour is as $t \rightarrow \infty$. What is $\lim_{\omega \rightarrow 1} y(t)$ of your solution?

- (10) Write the equation for the plane going through the three points $A = (1, 0, -1)$, $B = (1, -2, 4)$ and $C = (2, 1, 3)$. Decide whether or not the plane intersects the z -axis and, if so, give the point of intersection.