

Sample Problems for the 2nd Midterm. These are problems from 222 midterms given in the past. Old exams are available on the web, but be warned that the syllabus changes every year, so that problems which were on the 2nd midterm 2 years ago may show up on this year's 1st midterm. Don't prepare for last year's midterm!

Look for similar problems in the problem section in the notes. Do not assume that the exam will be exactly like these problems. This is just a sample of what the level will be. The length of the problems does not correspond to the length of the problems in the exam.

- (1) (a) Find

$$\Re \left\{ \frac{e^{(1-i)x}}{1+2i} \right\}.$$

- (b) You are given an angle x whose Cosine and Sine are given by

$$\sin x = A, \text{ and } \cos x = \sqrt{1 - A^2}.$$

Compute $\cos 6x$.

- (2) Solve the following integral using complex numbers

$$\int \sin^2 3x \cos 5x dx$$

- (3) (a) Find the general solution of $\frac{dy}{dx} = (\cos y)^2(1+x)$.

- (b) Which solution satisfies $y(0) = \frac{\pi}{3}$?

- (4) (a) Find the solution of
$$\begin{cases} x \frac{dy}{dx} + 3y = 2 + x^2 \\ y(1) = B \end{cases}$$

- (b) For which value of B does the limit $\lim_{x \rightarrow 0} y(x)$ exist?

- (5) Find the general solution of $5 \frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 5y = 0$. For which solution(s) $y(x)$ of the equation does the limit $\lim_{x \rightarrow \infty} y(x)$ exist?

- (6) Find a particular solution of the equation

$$\frac{d^3 y}{dx^3} - 5 \frac{dy^2}{dx^2} + \frac{dy}{dx} + 3y = x + \sin x.$$

- (7) The equation for the displacement $y(t)$ of a spring subject to a forced vibration of frequency ω is

$$\frac{dy^2}{dx^2} + 2 \frac{dy}{dx} + 2y = \sin(\omega t).$$

Are we considering friction? Find the solution for $\omega \neq 1$ and describe what the behaviour is as $t \rightarrow \infty$. What is $\lim_{\omega \rightarrow 1} y(t)$ of your solution?

- (8) Write the equation for the plane going through the three points $A = (1, 0, -1)$, $B = (1, -2, 4)$ and $C = (2, 1, 3)$. Decide whether or not the plane intersects the z -axis and, if so, give the point of intersection.