Sample Problems for the Final. Look for similar problems in the problem section in the notes. Do not assume that the exam will be exactly like these problems. This is just a sample of what the level will be. The length of the problems does not correspond to the length of the problems in the exam. These problems cover only the last part of the course. You can use the previous sample problems for the material covered in first and second midterms.

(1) Assume a force is being exerted on a particle lying on the plane

$$2x + y - 2z = 2$$

and assume the force is constant given by

$$f = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Find the normal to the plane.
(b) Decompose the force into a component perpendicular to the plane and another one parallel to the plane.

(2) We say that two curves intersect at an angle θ whenever the tangents to the curves at the point of intersection form an angle θ.

The following two curves intersect at $(1, 0)$.

$$x_1(t) = \begin{pmatrix} e^t \\ t^2 \end{pmatrix}, \qquad x_2(s) = \begin{pmatrix} e^s + 1 - e \\ \ln s \end{pmatrix}.$$

(a) Find the values of t and s for which the intersection takes place.
(b) Find the angle of intersection.
(c) Find the speed at the moment of intersection for each one of the curves.
(d) Are the acceleration vectors perpendicular at the point of intersection?

(3) Let $A = (2, 1, 0)$, $B = (1, 0, -1)$, $C = (0, 1, 1)$ be three points.
(a) Draw a picture of the three points.
(b) Calculate the normal to the plane that goes through the three points.
(c) Find the equation of the plane using the normal.
(d) Find the parametric equation of the plane.
(e) Find the area of the triangle ABC.
(f) Find the distance from $(0, 0, 0)$ to the plane.
(g) Find the intersection of the plane with the three axis.

(4) Given the points $A = (1, 0, -2)$ and $B = (0, 3, -1)$.
(a) Does the line that goes through the points intersect the z-axis? If so, which one is the point of intersection?
(b) Find the cosine of the angle that the line makes with the z-axis.

(5) Graph the parametrized curve given by

$$x(t) = \begin{pmatrix} t^2 \\ \cos t \end{pmatrix}$$

and find all the points for which the tangent is vertical or horizontal.

(6) Consider the polar graph $r = \theta e^{-\theta}$.
(a) Find an integral for the length of the curve from $\theta = 0$ to $\theta = \pi$
(b) Find the angle formed by the tangent and the radius at a value θ_0.
(c) Sketch the polar graph.