Section 11.4 or 11.5

This will be updated later in the week. I'm too busy right now, but the quiz wont cover the stuff I promised to write up but left out of this file.

1 Problem from Class

I dont know remember which problem this was, but we were trying to determine whether whether the following sequence converges or diverges:

$$\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n.$$

The idea is to show that it diverges by the nth-term test. This test says that if the terms of the *sequence* dont converge to zero, then the *series* diverges. This is very important. Ask me for help if you dont understand the difference between the sequence and the series defined by it.

To show that the terms dont go to zero, we will employ a different method from the one that professor Dickey uses in class. I think this is slightly easier, but take your pick for the test. We will use the fact that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. We proceed as follows:

$$\begin{aligned}
\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^n &= \lim_{n\to\infty}\left(\frac{n}{n}-\frac{3}{n}\right)^n \\
&= \lim_{n\to\infty}\left(\frac{n-3}{n}\right)^n \\
&= \lim_{n\to\infty}\left(\frac{n}{n-3}\right)^{-n} \\
&= \lim_{n\to\infty}\left(\frac{n-3+3}{n-3}\right)^{-n} \\
&= \lim_{n\to\infty}\left(1+\frac{3}{n-3}\right)^{-n} \\
&= \lim_{n\to\infty}\left(1+\frac{1}{\frac{n-3}{3}}\right)^{-n}
\end{aligned}$$

Now we make the substitution $m = \frac{n-3}{3}$ (so $n = 3m + 3$). Note that as $n \to \infty$, $m \to \infty$ as well. Thus,

$$\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n-3}{3}}\right)^{-n}$$

$$
\begin{aligned}
&= \lim_{n\to\infty}\left(1+\frac{1}{m}\right)^{-(3m+3)}\\
&= \lim_{n\to\infty}\left(1+\frac{1}{m}\right)^{-3m-3}\\
&= \lim_{n\to\infty}\left(1+\frac{1}{m}\right)^{-3m}\cdot\left(1+\frac{1}{m}\right)^{-3}\\
&= \lim_{n\to\infty}\left(\left(1+\frac{1}{m}\right)^{m}\right)^{-3}\cdot\left(1+\frac{1}{m}\right)^{-3}\\
&= e^{-3}\cdot 1 = e^{-3}
\end{aligned}
$$

Since $e^{-3} \neq 0$, the series diverges. I'll add more later. Let me know if you have questions, and I can explain this part better as well.