

UNIVERSITY OF WISCONSIN, DEPARTMENT OF MATHEMATICS
2009 DISTINGUISHED LECTURE SERIES

SHOUWU ZHANG

Lecture 1: Rational points on curves, Monday, November 30 (2009), 4-5pm, Van Vleck B239.

Lecture 2: Gross-Schoen cycles and dualising sheaves, Tuesday, December 1 (2009), 4-5pm, Van Vleck B239.

Lecture 3: Gross-Schoen cycles and triple product L-series, Wednesday, December 2 (2009), 4-5pm, Van Vleck B239.

The first lecture is a public lecture intended to very general audience. The second and third are colloquium type lectures.

Shouwu Zhang is a professor of Mathematics at the Columbia University and L.-K. Hua Chair Professor, Chinese Academy of Sciences, China. He is well-known for his vast generalization of the deep and beautiful Gross-Zagier formula, sometimes called Gross-Zagier-Zhang formula, and for his proof of the Bogomolov conjecture in arithmetic geometry. He got his PhD in 1991 at Columbia University under the guidance of Lucien Szpiro (and also Gerd Faltings). He was an invited ICM speaker at Berlin in 1998 and was awarded a Morningside Gold Medal of Mathematics in the same year by the International Congress of Chinese Mathematicians. He was a Chang-Jiang Chair Professor, Tsinghua University, China (2000-03), a Clay Foundation Prize Fellow (2003), and a Guggenheim Fellow (2009).

Abstract: Lecture 1: The structure of rational solutions to a polynomial equation depends on the structure of corresponding algebraic variety. In case of a curve of genus zero, the problem of finding all solutions can be completely solved using Hasse-Minkowski principle. In case of genus one, the obstruction to the Hasse-Minkowski principle is conjectured to be finite; and the set of rational points is a finitely generated group by the Mordell-Weil theorem if it not empty. In case of genus two or bigger, the set of solutions is finite by Faltings theorem.

A major unsolved problem today is the effectivity of solutions for curves of genus one or bigger. For elliptic curves, one has the Birch and Swinnerton-Dyer (BSD) conjecture which relates the Mordell-Weil group and the central values of L-series arising from counting rational points over finite fields. For curves of genus two or bigger, one has the ABC conjecture and its refinements providing some effective bounds for curves. In function field case, these conjectures are consequences of Bogomolov-Miyaoka-Yau.

Lectures 2 and 3: Gross and Schoen have constructed a cohomologically trivial 1-cycle on the triple product of a curve by a modification of the diagonal cycle. In lecture 2, he

will explain his recent formula for the height of this cycle in term of relative dualising sheaf. He will also explain the applications of this formula to ABC conjecture, Bogomolov Conjecture, and Tate's conjecture for variety over finite fields.

Gross and Kudla have conjectured a formula to related the height of Gross–Schoen cycles on Shimura curves and the central derivatives of the triple product L-series for triples of newforms of weight two. This conjecture is proved in his joint work with Xinyi Yuan and Wei Zhang with a great generality. In Lecture 3, Shouwu will explain this formula and its applications to rational points to elliptic curves.