Solution Sets

Consider the following problems, keeping in mind your work in 135 on what impact different operations have on changing the solution set of an equation.

1. Compare the solution sets of the two equations $3x + 7 = -2x + 22$ and $x = 3$.
2. (a) Are the solution sets to the two equations $71x^4 - 3\sqrt{x} + \frac{2}{x} = 4$ and $x = 1$ the same? How do you know?

   (b) What can you say about the solution sets of the two equations $71x^4 - 3\sqrt{x} + \frac{2}{x} = 70$ and $x = 1$?
3. (a) What is the solution set of the equation $a + 8 = 8 + a$?

   (b) What might be a reasonable solution set for the equation $1 = 1$?

   (c) Two equations are called **equivalent** if they have the same solution set. Discuss if you think the equations $2(a + 1) = 2a + 2$ and $1 = 1$ are equivalent.

   (d) Are the equations $3y + (5 - y) = 2(y + 2)$ and $0 = 1$ equivalent? Can you think of some other equations equivalent to $0 = 1$?
4. Find the solution set of $x(2x - 1) = 0$. What might be a common pitfall for students?
5. Compare the graphs of the two equations $y = -3x + 5$ and $y = \frac{(x-2)(-3x+5)}{x-2}$.
6. Evaluate the student's reasoning in the following argument. A step by step analysis might be helpful.

   Suppose that $a = b$. That's OK, right? I mean it's true for some choices of a and b, right? But something funny happens...

$$\begin{aligned} a &= b \\ a + a &= a + b \\ 2a &= a + b \\ 2a - 2b &= a + b - 2b \\ 2(a - b) &= a + b - 2b \\ 2(a - b) &= a - b \\ 2 &= 1 \end{aligned}$$

   I think all the steps are right... Even when I start out with a and b being equal, I end up with something that doesn't make sense. What's going on?

7. (a) Evaluate the student's reasoning in the following argument. It may help you to figure out what happens at each step.

Suppose I know that $a^2 = b^2$, and I want to show that $x - a$ equals $x + b$. Let's see what happens:

$$\begin{aligned} x - a &= x + b \\ (x-a)^2 &= (x+b)^2 \\ x^2 - 2a + a^2 &= x^2 + 2b + b^2 \\ -2a + a^2 &= 2b + b^2 \\ -2a &= 2b \\ (-2a)^2 &= (2b)^2 \\ 4a^2 &= 4b^2 \\ a^2 &= b^2 \end{aligned}$$

We know the last line to be true, so therefore $x - a = x + b$.

(b) In light of your work in part 7a, evaluate the student's reasoning in the following argument.

I want to show that $\frac{a}{a+1} + \frac{3}{a+2} + \frac{3}{(a+1)(a+2)}$ equals $\frac{a+3}{a+1}$.

$$\begin{aligned} \frac{a}{a+1} + \frac{3}{a+2} + \frac{3}{(a+1)(a+2)} &= \frac{a+3}{a+1} \\ \frac{a}{a+1}\left(\frac{a+2}{a+2}\right) + \frac{3}{a+2}\left(\frac{a+1}{a+1}\right) + \frac{3}{(a+1)(a+2)} &= \frac{a+3}{a+1}\left(\frac{a+2}{a+2}\right) \\ \frac{a^2+2a}{(a+1)(a+2)} + \frac{3a+3}{(a+1)(a+2)} + \frac{3}{(a+1)(a+2)} &= \frac{a^2+5a+6}{(a+1)(a+2)} \\ \frac{a^2+5a+6}{(a+1)(a+2)} &= \frac{a^2+5a+6}{(a+1)(a+2)} \end{aligned}$$

We know that's true, so we must know that $\frac{a}{a+1} + \frac{3}{a+2} + \frac{3}{(a+1)(a+2)}$ equals $\frac{a+3}{a+1}$.

(c) Can you think of a clearer way for the student in part 7b to make their argument?